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FINAL REPORT

(3rd year of the Project)

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Attempt to Establish an Atmospheric Model of the Nontropical Latitudes on the Basis of Simple Suppositions

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I Introduction

Chapter 1: The Suppositions

The manifestations of the atmospheric phenomena and processes are so various that they have long before created the need for the establishment of an atmospheric model of the greatest possible comprehensiveness. It is obvious that such a model is the better, the more empirical facts can later be incorporated in it, and the less forcibly this can be done on a physical basis and causal finishs. It must be flexible in accordance with the variety of the atmospheric phenomena, that is to say, it must allow of certain variations to be made to it, without, however, being superficial.

The aim of this analysis is to set up a model of the atmosphere which is applicable to mean conditions, and to offer an initial work of this kind. Comparison with empirical data may prove how far this effort has been successful.

In deducing the model no use whatever has, intentionally, been made of empirical meteorology. This has proved necessary in order to be able to recognize the physical causality of the development of the individual layers in the atmosphere. Only the simplest mathematical and physical relations have been employed and only the quantities, being given by the earth as a planet, that have been used — though not each time individually named — are those stated below. They have been designated „basic facts".

Acceleration due to gravity g

Velocity of the earth's rotation ω

Mass of the atmosphere

Inclination of the earth's axis to the earth's orbit and the resulting radiative conditions

The existence of special atmospheric layers where the sun's radiation is absorbed; particularly, the existence of the upper border of the ozone layer.

Surface friction.

Other author's results have not been used unless these were attained on a strictly theroretical basis. Since empirical facts have not been employed, it has proved necessary now and then to use working hypotheses with a view to permit certain conclusions to be reached. However, it will turn out in each such case whether the hypothesis used has been correct or not. In deriving these initial conclusions, their materialization in nature must not be asked for.

We have marked only such conclusions as have found to be reliable upon thorough examination. These conclusions are indented, put in italics, and marked by double lines in the left margin. They are denoted by the letter "C" and bear serial numbers.

The analysis has been confined to the nontropical latitudes so as to avoid the range of investigation becoming too large. An analogous model of the tropical latitudes is left for future establishment.

The calculations and investigations of this analysis have been made in the (x, y, z)-system, and not in the (x, y, p)-system, nowadays common in meteorology. As later sections will show, for this study, the z-system is the more advantageous of the two. In the system used here, the x-axis points to the east, the y-axis to the north and the z-axis to the zenith.

The gradients have been founded on their mathematical definition, and therefore show in the direction of the highest increase of the values. Wherever disturbances are involved — which generally are not independent of the longitudes — the system of coordinates applied is oriented to the horizontal flow. In this, s designates the coordinate having the direction of the flow, while the n-axis is directed in such a way that it forms together with the coordinate axes of s and z, a mathematical rectangular system. The symbols used denote the following:

p	air pressure
T	air temperature
ϱ	air density
R	gas constant for dry air
g	acceleration due to gravity
ω	velocity of the earth's rotation
φ	geographic latitude
f	Coriolis parameter $2\,\omega \sin \varphi$
l	Coriolis parameter $2\,\omega \cos \varphi$
v_x	wind component in the direction of x
v_y	wind component in the direction of y
v_z	vertical air motion
∇	threedimensional gradient
∇_h	horizontal gradient
t	time

Since their investigation is to be confined to the basic facts as a working basis, the authors have been forced to utilize a theorem which, a priori, must be regarded valid as long as no contradictory results are reached:

„No meteorological quantity can grow beyond all limits". The authors are well aware of the makeshifty nature of this theorem which they are obliged to use due to their adherence to merely the basic facts.

Particular importance has been attached to the layers of the maximum and vanishing meridional gradients of the three atmospheric parameters, p, T and ϱ, which are connected with one another by the gas equation. For shortness, those layers have been designated in a way suggested by E. Müller, viz., incorporating the Greek terms „homos" (denoting one and the same, like), and „pleistos" (denoting most, the largest quantity).

H_p Layer of vanishing meridional pressure gradient = homobaric layer

P_p Layer of maximum meridional pressure gradient = pleistobaric layer

H_T Layer of vanishing meridional temperature gradient = homothermic layer

P_T ·Layer of maximum meridional temperature gradient = pleistothermic layer

H_ϱ Layer of vanishing meridional density gradient – homopycnic layer

P_ϱ Layer of maximum meridional density gradient = pleistopycnic layer.

First, only a decrease in temperature from the low to the high latitudes, caused by differing radiation is assumed to exist on the earth's surface. To this basic fact a number of physical laws in the form of mathematic equations and qualitative considerations are applied. These are the equations used:

$$\frac{\partial p}{\partial z} = -g\varrho \tag{1}$$

(hydrostatic equation)

$$p = RT\varrho \tag{2}$$

(gas equation)

$$v_x = -\frac{1}{f\varrho}\frac{\partial p}{\partial y} \tag{3}$$

(equation for the geostrophic wind)

The latter equation represents a wind that is directed parallel to the isobares, that is, the Coriolis force and the force of the pressure gradient are in a state of equilibrium. Thus, there is no wind component that crosses the isobares, and, consistently, no transportation of air masses takes place between the areas of high pressure and low pressure. The geostrophic wind is a fictive wind, now known, to occur in nature very rarely. But as the wind component that is parallel to the isobares predominates over the wind component that is perpendicular to the isobares, the geostrophic wind represents the real wind conditions with a high degree of accuracy.

Further, use has been made of the Eulerian equations of motion.

$$\text{(4 a)} \quad \frac{dv_x}{dt} - f v_y + l v_z = -\frac{1}{\varrho}\frac{\partial p}{\partial x}$$

$$\text{b)} \quad \frac{dv_y}{dt} + f v_x = -\frac{1}{\varrho}\frac{\partial p}{\partial y}$$

$$\text{c)} \quad \frac{dv_z}{dt} = -\frac{1}{\varrho}\frac{\partial p}{\partial z} - g$$

For motion without acceleration, eq. (1) results from eq. (4c), and eq. (3) from eq. (4b).

This is the socalled Eulerian expansion:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z} = \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla \tag{5}$$

Differentiation with respect to z of the gas equation renders

$$\frac{1}{R}\frac{\partial p}{\partial z} = \varrho\frac{\partial T}{\partial z} + T\frac{\partial \varrho}{\partial z} \tag{6}$$

and differentiation with respect to y renders the meridional gradients

$$\frac{1}{R}\frac{\partial p}{\partial y} = \varrho\frac{\partial T}{\partial y} + T\frac{\partial \varrho}{\partial y} \tag{7}$$

In establishing the model, the determination of certain laws has proved necessary from the beginning. The model includes the deduction of various layers of marked characteristics (extrem or vanishing value) of the three quantities, pressure, temperature and density, as well as of the wind. The situation at a specific altitude of a layer having the extrem value of the characteristic „C" frequently brings about the question as to at what altitude, above or below, the layer showing the extrem value of the characteristic $\lambda(z)\,C$ is situated; $\lambda(z)$ being a factor dependent on the altitude. E. g. there is a layer showing the maximum condition of the meridional pressure gradient $\frac{\partial p}{\partial y}$; then, the problem is to find the altitude of the maximum of $\frac{1}{\varrho}\frac{\partial p}{\partial y}$. This has led to the compilation in the next chapter of a number of preliminary notes regarding those laws.

In view of the considerable amount of calculation that would have been required to ascertain the absolute heights of the individual layers, the authors have desisted from stating those. They have confined themselves to explaining only the relative altitudes of the individual layers, that is, the succession of these. Although the considerations involved have often been rather difficult, the authors' aim throughout has been physical and mathematical correctness as well as logical proceeding. They ask the reader's indulgence in case any paralogisms may have occurred. However, they should be much obliged if their considerations were used, and continued, by colleagues.

Chapter 2: Derivation of Relative Positions of Significant Meteorological Levels

In the investigation of pleisto-layers the principal point of interest is the relative positions of the individual layers of extreme values. As, mostly, either of the two factor functions is monotonous (e. g. monotonously decreasing), the question can be given a specific form:

$$f(z) = \lambda(z) \cdot g(z)$$

It is assumed that

$$\frac{d}{dz}\lambda(z) \gtrless 0 \text{ monotonously;} \quad \lambda(z),\ g(z) > 0$$

This assumption is justified for the reason that in our consideration, the accent is put on the mathematical extremes of the value functions rather than on those of the functions with regard to the signs. We want to determine the extreme values, disregarding the signs, of a function

$$f(z) = \lambda^*(z) \cdot g^*(z) \qquad (f,\ \lambda^*,\ g^* \gtrless 0)$$

Then, the problem can be reduced to that mentioned above if we assume that

$$|f^*(z)| = f(z)$$
$$|\lambda^*(z)| = \lambda(z)$$
$$|g^*(z)| = g(z)$$

These are the conditions of a maximum of the vertical distribution of $g(z)$:

$$\frac{d}{dz} g(z) = 0 \quad \text{and} \quad \frac{d^2}{dz^2} g(z) < 0$$

z^* be the height of this maximum; z^{**} the required height where $f(z)$ has a maximum. Here,

$$\frac{d}{dz} f(z)\Big|_{z^{**}} = \lambda(z)\frac{d}{dz} g(z) + g(z)\frac{d}{dz}\lambda(z) = 0$$

If $\lambda(z)$ be a monotonously increasing function (e. g. $1/\varrho$), $d\,f(z)/dz = 0$ can occur only above z^* for the reason that below this height, both terms at the right-hand side of the latter equation have the same sign. The opposite conditions hold good for a monotonously decreasing function $\lambda(z)$ (e. g. ϱ).

Conditions corresponding to the above can be assumed to exist in the case of a minimum, except that here, $d^2 g(z)/dz^2 > 0$.

C. I **A monotonously increasing (decreasing) factor function ($\lambda(z)$) causes the maximum (minimum) of a product function ($f(z)$) to be situated above the height of the maximum (minimum) of the other factor function ($g(z)$). In the case of "increasing" and "decreasing" changing their parts, the extreme of $f(z)$ is situated below the extreme of $g(z)$.**

Infinitesimal evaluation of course does not provide for the determination of the real values of the difference $(z^{**} - z^*)$. Theoretically, this can be infinite. It is dependent on the respective characters of the two factor functions.

*

Some figures in this study are of an unusual appearance which also need explanation.

An α-level be defined as a level of constant values of α, (α being p, T or ϱ). The basis of consideration be the (y, z)-plane that is, the meridional cross-section. That p-, T-, and ϱ-level which passes through a specific point P in the (y, z)-plane, be intersected by this plane. The tangents at point P to these three intersectional curves be designated "tracing tangents". The absolute and relative slopes of these three tracing tangents (Fig. 1), permit a number of conclusions to be reached. The conclusions resulting from drawing the three tracing tangents at several consecutive points P_i of the z-axis are typical of the physical vertical structure of an atmosphere system. Any meteorological laws that are required to draw the tracing tangents must be observed in order not to obtain conditions that would be physically impossible.

First, the laws of the relative positions of the tracing tangents of p, T and ϱ at point P will be deduced. In this deduction, a theoretic study made by *E. Müller* at the request of the authors will be used.

As p, T and ϱ are connected with one another by the eqs. (1) and (2), it is apparent that the tracing tangents cannot be drawn at choice, but that the slope of any of them rather depends upon the slopes of the other two.

α be any of the three parameters p, T and ϱ. We are now interested in the condition of an air column that would render the slope of the tracing tangent of α to be situated at point P between the slopes of the other two tracing tangents. The condition both necessary and sufficient for this is that in the α-coordinate system (α being the vertical coordinate), the signs of the slopes of the other two tracing tangents are opposite.

The relative slope of a level of constant β-values to one of constant α-values is represented by

$$\frac{\partial \alpha}{\partial y}\Big|_\beta = -\frac{\dfrac{\partial \beta}{\partial y}\Big|_\alpha}{\dfrac{\partial \beta}{\partial \alpha}}$$

First, ϱ be substituted for α.

In the ϱ-system,

$$-\frac{\left.\frac{\partial p}{\partial y}\right|_{\varrho}}{\frac{\partial p}{\partial \varrho}} = -\frac{\left.\varrho R \frac{\partial T}{\partial y}\right|_{\varrho}}{RT + \varrho R \frac{\partial T}{\partial \varrho}}$$

$$= -\frac{\left.\frac{\partial T}{\partial y}\right|_{\varrho}}{\frac{\partial T}{\partial \varrho}} \frac{1}{\left(1 + \frac{T}{\varrho\, \partial T / \partial \varrho}\right)}$$

The left-hand side of this equation and the fraction preceding the parenthesis represent the slopes of the tracing tangents of p and T respectively. To render these opposite to one another the term within the parenthesis must be negative, i. e.

$$1 + \frac{T}{\varrho \frac{\partial T}{\partial \varrho}} < 0 \text{ or } 1 + \frac{T}{\varrho} \frac{\frac{\partial \varrho}{\partial z}}{\frac{\partial T}{\partial z}} =$$

$$= \frac{1}{\frac{\partial T}{\partial z}} \left(\frac{\partial T}{\partial z} - \frac{g}{R} - \frac{\partial T}{\partial z} \right) < 0$$

Since g/R is positive, it yields: $\frac{\partial T}{\partial z} > 0.$

C. II **In the case of the temperature increasing with altitude, the tracing tangent of density is the middle one between those pertaining to P.**

Now, α be replaced by p; then the corresponding equations are as follows:

$$-\frac{\left.\frac{\partial \varrho}{\partial y}\right|_{p}}{\frac{\partial \varrho}{\partial p}} = \frac{\left.\frac{p}{RT^2} \frac{\partial T}{\partial y}\right|_{p}}{\frac{1}{RT} - \frac{p}{RT^2} \frac{\partial T}{\partial p}}$$

$$= -\frac{\left.\frac{\partial T}{\partial y}\right|_{p}}{\frac{\partial T}{\partial p}} \frac{1}{\left(1 - \frac{T}{p\, \partial T / \partial p}\right)}$$

Again, the parenthesis must be negative:

$$1 - \frac{T}{p \frac{\partial T}{\partial p}} < 0$$

$$1 - \frac{T}{p} \frac{\frac{\partial p}{\partial z}}{\frac{\partial T}{\partial z}} = 1 + \frac{g \varrho T}{p \frac{\partial T}{\partial z}}$$

$$= \frac{1}{\frac{\partial T}{\partial z}} \left(\frac{\partial T}{\partial z} + \frac{g}{R} \right) < 0$$

The right-hand side cannot be negative unless, first, the factor $\frac{1}{\partial T / \partial z}$ preceding the parenthesis is negative and, second, the parenthesis itself remains positive. This renders

$$-\frac{g}{R} < \frac{\partial T}{\partial z} < 0.$$

C. III **In the case of the temperature decreasing with altitude — this decrease however being lesser than what would correspond to the vertical temperature gradient in the homogeneous atmosphere $(-g/R)$ — the tracing tangent of pressure is the middle one between those pertaining to P.**

Finally, α be substituted by the temperature.

$$-\frac{\left.\frac{\partial p}{\partial y}\right|_{T}}{\frac{\partial p}{\partial T}} = -\frac{\left.RT \frac{\partial \varrho}{\partial y}\right|_{T}}{R \varrho + RT \frac{\partial \varrho}{\partial T}}$$

$$= -\frac{\left.\frac{\partial \varrho}{\partial y}\right|_{T}}{\frac{\partial \varrho}{\partial T}} \frac{1}{\left(1 + \frac{\varrho}{T\, \partial \varrho / \partial T}\right)}$$

The equation of condition in respect of the negative parenthesis now renders

$$1 + \frac{\varrho}{T \frac{\partial \varrho}{\partial T}} = 1 + \frac{\varrho}{T} \frac{\frac{\partial T}{\partial z}}{\frac{\partial \varrho}{\partial z}}$$

$$= \frac{\frac{g}{R} - \frac{\partial T}{\partial z} + \frac{\partial T}{\partial z}}{\frac{g}{R} + \frac{\partial T}{\partial z}} < 0$$

This is not possible unless

$$\frac{\partial T}{\partial z} < -\frac{g}{R}.$$

C. IV **In the case of the temperature decreasing with height stronger than in the homogeneous atmosphere, the tracing tangent of temperature is the middle one among the three pertaining to P.**

The latter case is not applicable in nature; for if this would be so, air density would have to increase with height. In the case of the earth's surface being the only heating level, the condition always is $0 > \frac{\partial T}{\partial z} > -\frac{g}{R}$. That means, in the respective figures, the level of p must always be situated between the two others.

In Figures 2, 6, 9, 13, the projections upon the (y, z)-plane of the vectors ∇p, ∇T and $\nabla \varrho$ at point P_i are represented by arrows which are perpendicular to the tracing tangents. Because of the omission of quantitative considerations, the lengths of those arrows do not matter; it is only their relative directions as to one another, and as to the z-axis, that do count.

The arrows point in the direction of growing values. An arrow pointing to the lower left thus indicates an increase of the respective parameter downwards and towards lower latitudes. Emphasis is put on those levels in which any of the arrows coincides with the z-axis. thus indicating a level of vanishing values of the meridional gradient.

Part a of the Figures 9 and 13 illustrates the vertical distribution of $\frac{\partial p}{\partial y}$, $\frac{\partial T}{\partial y}$, $\frac{\partial \varrho}{\partial y}$, (which for shortness are in our study called „meridional gradients"), though not the real value, but one multiplied by some factor. Eq. (7)

$$\frac{1}{R} \frac{\partial p}{\partial y} = \varrho \frac{\partial T}{\partial y} + T \frac{\partial \varrho}{\partial y}$$

furnishes a relationship which must be taken into account at any altitude when plotting the figures. Yet this relationship facilitates rather than impairs the drawing of the lines. E. g. if the p-curve passes through zero, $(\partial p / \partial y = 0)$, the T- and ϱ-curves must at this height be at an equal distance in opposite directions, from the z-axis. In the case of the p-curve intersecting any of the other two curves, the remaining one must pass through zero. In order that the aid offered by those "laws" may not be neglected, the factors of eq. (7) have been retained. The disadvantage of doing so is that the layers of extreme values of the products change their altitudes as against the layers of extreme gradients. However, this does not matter greatly since the absolute heights have been paid no attention to.

Since, considering the rapid decrease of p and ϱ with height, the distances from the z-axis of the curves would be very small in the upper parts of the figures, the curves have been plotted in such a way as though all the gradients had once more been multiplied by $1/\varrho$. The curves, thus, represent the approximate vertical distribution of $\frac{1}{R\varrho} \frac{\partial p}{\partial y}$, $\frac{\partial T}{\partial y}$, $\frac{T}{\varrho} \frac{\partial \varrho}{\partial y}$.

The Figures 9, 13, 21, 18, 19 show the appertaining system of circulation, the branches of same consisting of the nongeostrophic horizontal and of the vertical movements.

In the colored representation, the symbols in respect of pressure, temperature and density are shown as follows:

pressure	yellow
temperature	red
density	blue

In the one-color representation, these are illustrated as follows:

pressure	solid line
temperature	dotted line
density	dashed line

II The Earth's Surface as a Heating Level

Chapter 3: The Homopycnic Layer and the Wagner-Linke-Layer

It is known that on the surface of the earth there is a temperature gradient that is directed from north to south, i. e., of negative sign. Assuming the earth's surface to be the only heating level, the conclusion that can so far be drawn is that its effect will decrease with height and that at a certain, very high altitude, the respective meridional gradients of pressure, temperature and density will be zero.

No statement can be made as yet regarding the meridional pressure gradient that exists on the earth's surface, $\partial p_0 / \partial y$; what we do know, is the situation at higher levels. The stronger radiation that prevails in lower latitudes causes the constant pressure levels to rise. With increase in height the pressure difference between low and high latitudes will rise; its increase weakening as the height increases, since according to the basic facts, the difference in temperature between the high and the low latitudes decreases with height. This pressure gradient above the surface of the earth causes west wind as per eq. (3).

Earlier meteorologic papers try to explain the development of a low-pressure area in this way: A certain part of the earth's surface is subject to intensified radiation, so the constant pressure levels will rise, with the air aloft flowing off this 'air mountain', and the resulting mass deficiency consistently causing the pressure on the earth's surface to drop.

No consideration has in that theory been given to the Coriolis force, which holds together the air pressure systems and thus does not allow the air aloft to flow off towards lower pressure. It is not clarified in how far these conditions apply to the equatorial regions where the Coriolis force is non-existent, and also for this reason the deduced model is limited to the nontropical latitudes.

It seems necessary in this connection to raise the problem of the possibility of levels of extreme or disappearing gradients of the meteorological parameters developing at any altitude. This requires the signs of the surface gradients from eq. (7) to be evaluated. For it is its sign near the surface that determines to which extreme condition — maximum or minimum — any of those gradients may go at a specified altitude.

$\partial T_0 / \partial y$ is assumed to be negative. No statement can so far be made relative to the pressure gradient on the earth's surface, $\partial p_0 / \partial y$. As long as no result can be obtained that would render $\partial p_0 / \partial y \neq 0$, the hypothesis used hereunder is $\partial p_0 / \partial y = 0$. So, due to eq. (7), $\partial \varrho / \partial y$ must be positive on the earth's surface.

The statement — so far not definitely proved — that air density on the earth's surface is increasing poleward from the lower latitudes allows an important conclusion to be drawn if considered in connection with eq. (1).

There must be a specific height at which the meridional gradient of density fades. This shall be worked out.

From eq. (1), (2) and (6) the following can be derived:

$$\frac{\partial \varrho}{\partial z} = -\frac{\varrho}{T}\left(\frac{g}{R} + \gamma\right) \tag{8}$$

where γ = vertical temperature gradient $\partial T / \partial z$, to be independent of z.

Assuming a constant mean temperature T_m (an operation analogous to the finding of the barometric height formula), the foregoing equation can be integrated as follows:

$$\ln \varrho = \ln \varrho_0 - \frac{1}{T_m}\left(\frac{g}{R} + \gamma\right) z \tag{9}$$

where ϱ_0 = air density on the earth's surface.

Assigning to the variables of this equation subscript 1 for the warm air in the south and subscript 2 for the cold air in the north, and claiming the densities of warm air and cold air to be equal at the required altitude $z = h$, $\ln \varrho_1 = \ln \varrho_2$, the result is

$$z = h = \frac{\ln \dfrac{\varrho_{01}}{\varrho_{02}}}{\dfrac{1}{T_{m_1}}\left(\dfrac{g}{R} + \gamma_1\right) - \dfrac{1}{T_{m_2}}\left(\dfrac{g}{R} + \gamma_2\right)} \tag{10}$$

This is the altitude of the layer of a theoretically vanishing meridional density gradient (homopynic layer), and its value is positive on the assumption that ϱ_0 is larger in high latitudes than in low.

In 1919 *F. Linke* [1] derived a formula for a layer of theoretically vanishing changes of density, which had been found already 9 years earlier by *A. Wagner* when he studied the height of the maximum interdiurnal pressure changes [2]. *Linke*'s suppositions were:

$$\frac{\partial p_0}{\partial t} = 0; \quad T = T_m + \frac{\gamma}{2} h^*$$

(where T_m = mean temperature between surface and height h^*, $\gamma = \partial T / \partial z$).

With $\partial \varrho_0 / \partial T_m$ equal to 0, he found the height of the required layer to be

$$h^* = \frac{R\, T_m^2}{g\, T}.$$

It is desirable to ascertain the relationship that holds between eq. (10) and that derived by Linke.

In *Linke*'s derivation, p_0 is assumed to be constant, i. e.

$$\frac{\varrho_{01}}{\varrho_{02}} = \frac{T_{02}}{T_{01}}.$$

When using the Δ-symbol (showing the difference) instead of the subscripts 1 and 2, eq. (10) results to be

$$\text{(10 a)} \quad h = \frac{\ln \left(1 + \dfrac{\Delta T_o}{T_o} \right)}{\dfrac{1}{T_m} \left(\dfrac{g}{R} + \gamma \right) - \dfrac{1}{T_m + \Delta T_m} \left(\dfrac{g}{R} + \gamma + \Delta \gamma \right)}$$

The logarithm is developed in a series and discontinued after the 1st term. Assuming $\Delta\gamma$ to be equal to 0:

$$\text{(10 b)} \quad h = \frac{\dfrac{\Delta T_0}{T_0}}{\dfrac{1}{T_m} \left(\dfrac{g}{R} + \gamma \right) \left(1 - \dfrac{T_m}{T_m + \Delta T_m} \right)}$$

The last bottom factor can be transformed since

$$\frac{\Delta T_m}{T_m} \ll 1; \qquad 1 - \frac{1}{1 + \dfrac{\Delta T_m}{T_m}} \approx 1 - \left(1 - \frac{\Delta T_m}{T_m} \right)$$

Then it follows

$$h = \frac{\Delta T_o \cdot T_m}{T_o \left(\dfrac{g}{R} + \gamma \right) \dfrac{\Delta T_m}{T_m}}$$

With $\Delta\gamma = 0$, the result is $\Delta T_o = \Delta T_m$, and the following equation is obtained:

$$\text{(10 c)} \quad h = \frac{T_m^2}{T_o \left(\dfrac{g}{R} + \gamma \right)}$$

The latter is the form of the equation as derived by *Linke* originally [1], and expanded by γ. While eq. (10) is more complicated than *Linke*'s derivative, it is of a more comprehensive nature.

In a later publication [3] *Linke* called attention to a note by *F. M. Exner* saying that instead of using the mean temperature it would be more accurate to use the polytrope statement $T = T_o - \gamma z$ (γ = constant), which would render the height of the *Wagner-Linke*-layer to be

$$\text{(11)} \quad h^{**} = \frac{R T_o}{g}$$

that is, the height of the socalled homogeneous atmosphere, an imaginary atmosphere having constant air density with $\gamma = -3.4°\mathrm{C}/100\,\mathrm{m}$.

We feel, however, that particularly when comparing warm and cold air, it would be of havier consequence to adopt the same $\partial T / \partial z$ for both warm and cold air, than to use two mean temperatures, and we therefore have applied *Linke*'s 1st formula for comparison purposes.

From eq. (10) it appears that the lower (the higher) the homopycnic layer, the greater (the lesser) will be the differences between the mean temperatures of warm and cold air. The altitude of the layer is higher with absolutely higher temperatures (summer) than it is with absolutely lower temperatures (winter). Due to these two mutually independent influences on the height of the homopycnic layer, an altitude of minimum temporal

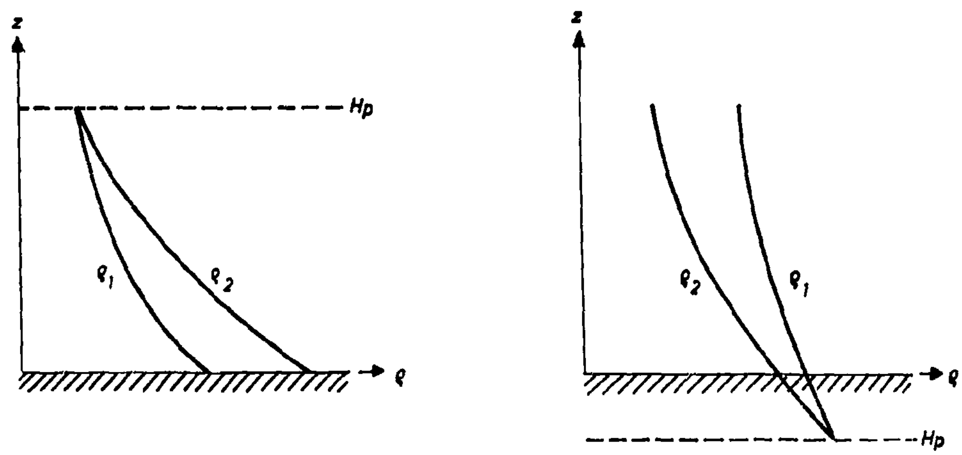


Fig. 1: Vertical decrease of density at high and low latitudes and the homopycnic layer

density variations will result in the temporal mean at a specified location.

If density near the earth's surface were lower in the polar region than in the low latitudes, i. e., if ϱ_{01} were greater than ϱ_{02}, $\ln \varrho_{01}/\varrho_{02}$ would be positive, with a negative altitude being obtained for the homopycnic layer (Fig. 1) because of $\frac{1}{Tm_1} - \frac{1}{Tm_2} < 0$

Thus, the homopycnic layer does not exist but on the assumption that density over the polar region is higher than over the low latitudes, which in turn arose from using statement $\partial p_0 / \partial y = 0$.

C. 1 a **If air density on the earth's surface is higher over the polar region than over the warmer, lower latitudes, there will be a layer of theoretically vanishing meridional density gradient (homopycnic layer), whose height is found by means of equation (10).**

This entails another conclusion, viz.,

C. 1 b **On the basis of the assumptions used here, the mean vertical decrease in air density is stronger over the polar region than over the lower latitudes (Fig. 1 a).**

density variations will result in the temporal mean at a specified location.

If density near the earth's surface were lower in the polar region than in the low latitudes, i. e., if ϱ_{01} were greater than ϱ_{02}, $\ln \varrho_{01}/\varrho_{02}$ would be positive, with a negative altitude being obtained for the homopycnic layer (Fig. 1) because of $\frac{1}{Tm_1} - \frac{1}{Tm_2} < 0$

Thus, the homopycnic layer does not exist but on the assumption that density over the polar region is higher than over the low latitudes, which in turn arose from using statement $\partial p_0 / \partial y = 0$.

C. 1 a **If air density on the earth's surface is higher over the polar region than over the warmer, lower latitudes, there will be a layer of theoretically vanishing meridional density gradient (homopycnic layer), whose height is found by means of equation (10).**

This entails another conclusion, viz.,

C. 1 b **On the basis of the assumptions used here, the mean vertical decrease in air density is stronger over the polar region than over the lower latitudes (Fig. 1).**

Chapter 4: The 'Null' Layer

The sign of $\partial p_0 / \partial y$ will now be considered, and a digression is necessary for this purpose. Our first approach is to find out the possibility of an extreme condition of $\partial p / \partial y$ occurring at a specified altitude. Such an extreme would have to be a maximum on the basis of eq. (1), according to which the decrease of pressure with height takes place at a faster rate in the heavier air over the high latitudes than over the low ones.

A maximum of the meridional pressure gradient in its vertical distribution is given by $\frac{\partial^2 p}{\partial z\, \partial y} = 0$, and using eq. (1) the following is obtained:

$$(12) \qquad \frac{\partial^2 p}{\partial z\, \partial y} = \frac{\partial^2 p}{\partial y\, \partial z} = -g \frac{\partial \varrho}{\partial y} \; (= 0)$$

$\partial \varrho / \partial y = 0$ representing the definition of the homopycnic layer (and frequently to be abbreviated $H\varrho$ in the following). This relation was shown earlier by *Doporto* [4] in his excellent study on the matter.

C. 2: **There is a layer of maximum meridional pressure gradient (pleistobaric layer) at the altitude of the homopycnic layer.**

Calculation shows these two layers to occupy exactly the same height. It must, however, be considered that the hydrostatic equation (1) used in the operation is correct only relative to movements without acceleration (left-hand side of eq. (4c) = 0). Consequently, there may in a particular case be an extremely small difference in height between the two layers which will be disregarded in the below analysis.

Derived from the pressure field, the gradientic wind must have an extreme value above the extreme of the pressure gradient. Differentiating eq. (3) with respect to z shows:

$$(13) \qquad \frac{\partial v_x}{\partial z} = -\frac{1}{f\varrho} \frac{\partial}{\partial z} \left(\frac{\partial p}{\partial y} \right) + \frac{1}{f\varrho^2} \frac{\partial p}{\partial y} \frac{\partial \varrho}{\partial z}$$

It appears that the height difference between the two layers is caused by the 2nd term of eq. (13), right-hand side. Above the layer of maximum (negative) $\partial p / \partial y$, the first term is minus, $\partial p / \partial y$ now decreasing with increasing height. Both $\partial \varrho / \partial z$ and $\partial p / \partial y$ being negative, the 2nd term remains positive in both cases. For an extreme of the gradientic wind, eq. (13) will become zero, this being possible only above the layer of maximum pressure gradient where both terms are of opposite signs. It would be thinkable indeed, that this layer of maximum winds will not be reached until very high altitudes of the atmosphere. But the following considerations show, that this layer has to be situated not too far away from the heating level: Based on the decisions hitherto the earth' surface represents a level of constant pressure, whereas the isothermic levels show the strongest slope there. With increasing altitude the slope of the p-levels increases, but that of the T-levels decreases.

Whereas the behaviour of the p-levels already have been explained at the beginning of chapter 3, it seems to be necessary to give a short consideration on the slope of T-levels. As is well known the slope of this level is given by

$$tg\alpha = -\frac{\partial T / \partial y}{\partial T / \partial z}$$

Now we want to regard the behaviour of the terms on the right side of this equation in their dependency on the height. On the earth' surface $\partial T / \partial y$ is determined by the basic facts. The action of the vertical „Austausch" (mixing of the air) aspires to a constant (negative) vertical temperature gradient above a fixed place. The value of this vertical temperature decrease mainly is a function of the temperature of the earth' surface. Consequently the vertical temperature gradient increases towards lower latitudes thus, with nearly constant local $\partial T / \partial z$, $\partial T / \partial y$ decreases with increasing altitude and therewith the slope of the temperature levels weakens. Due to the turn of the p- and T-level, not too far away from the heating level finally this levels will be parallel, which, as it will be shown later, represents a maximum of the zonal wind.

C. 3: **Above the layer of maximum meridional pressure gradient (pleistobaric layer) there must be a layer of maximum gradientic winds.**

In fact this conclusion is obvious from eq. (3) since $\varrho(z)$ decreases with height, and it is indicated also by the preliminary notes of Chapter 2.

Basing on the existence of a layer of maximum winds — when it is not yet determined whether or not the maximum of the real wind occurs at exactly the same altitude as that of the gradientic wind — other conclusions can be reached.

Use of eq. (12) transforms eq. (13) to

$$(13\,a) \qquad \frac{\partial v_x}{\partial z} = +\frac{1}{f\varrho^2} \left(\frac{\partial p}{\partial y} \frac{\partial \varrho}{\partial z} - \frac{\partial p}{\partial z} \frac{\partial \varrho}{\partial y} \right) = \frac{1}{f\varrho^2} (\nabla p \times \nabla \varrho)_{x\text{-comp.}}$$

Then, using eq. (14)

$$(14) \qquad \varrho (\nabla T \times \nabla p) = p (\nabla T \times \nabla \varrho) = T (\nabla p \times \nabla \varrho)$$

we obtain the expression as it has been shown earlier by *W. Attmannspacher* [5], viz.,

$$(15) \qquad \frac{\partial v_x}{\partial z} = \frac{R}{fp} (\nabla T \times \nabla p)_{x\text{-comp.}}$$

That is to say, the zonal wind (that along the x-axis) will reach an extreme value (here, a maximum) at that altitude at which the projections on the (y, z)-plane of the three-dimensional vectors of temperature, pressure and density are parallel. As has been shown in the above-cited study, the wind extreme is persistent only if the vectorial product of the three-dimensional gradients is zero. It is seen, therefore, that the layer of the persistent wind extreme is a barotropic (thermotropic and stereotropic) level, which result has been found also by G. *Hollmann* [6] independently of *W. Attmannspacher*.

This allows an essential conclusion to be drawn in respect of the large-scale vertical motion, and for this purpose, we shall briefly reproduce the considerations given in [5].

Practically adiabatic conditions can be assumed to occur in the free atmosphere. To deduce the large-scale vertical motion, the operation is started from the First Law of Thermodynamics:

$$\frac{dQ}{dt} = c_p \frac{dT}{dt} - \frac{1}{\varrho}\frac{dp}{dt} \tag{16}$$

where c_p = specific heat for p = *constant.*

For adiabatic $\left(\frac{dQ}{dt} = 0\right)$, stationary $\left(\frac{\partial T}{\partial t} = \frac{\partial p}{\partial t} = 0\right)$ conditions without friction eq. (16) is reduced to

$$\mathbf{v}\left(c_p \nabla T - \frac{1}{\varrho}\nabla p\right) = \mathbf{v}_h\left(c_p \nabla_h T - \frac{1}{\varrho}\nabla_h p\right) + v_z\left(c_p \frac{\partial T}{\partial z} - \frac{1}{\varrho}\frac{\partial p}{\partial z}\right) = 0 \tag{17}$$

Finally, this equation is solved for v_z, giving consideration to eq. (4):

$$v_z = -\frac{\frac{\partial T}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial T}{\partial y}\frac{\partial p}{\partial x}}{\varrho f\left(\frac{\partial T}{\partial z} + \frac{g}{c_p}\right)} = -\frac{(\nabla T \times \nabla p)_{z\text{-comp.}}}{\varrho f\left(\frac{\partial T}{\partial z} + \Gamma\right)} \quad \left(\Gamma = \frac{g}{c_p}\right) \tag{18}$$

When not disregarding lv_z in the x-component of *Euler's* equation, an additional term is obtained in the denominator of the foregoing eq. (18), which can be disregarded for nontropical latitudes.

Because of the parallelity of the three-dimensional gradients eq. (15) and (18) render the following, essential conclusion:

C. 4: **In the layer of the persistent wind maximum the largescale vertical motion is zero.**

The vectors ∇T and ∇p used by *W. Attmannspacher* will now be contemplated. ∇p (directed downward) stands nearly perpendicular because the vertical pressure gradient is much larger than the horizontal, and it tends slightly to the lower latitudes because $\partial p/\partial y$ is negative above the earth's surface. A somewhat stronger inclination of the temperature gradient, ∇T, to the same direction occurs as long as the wind increases with height. In the layer of vanishing large-scale vertical motion ∇T is parallel to ∇p, whereas below, it turns toward ∇p with increase in height. But since ∇p is not exactly perpendicular, the meridional temperature gradient cannot be zero where ∇T is parallel to ∇p. The conditions in the (y, z)-plane, which are representative of variations in the zonal wind, are suggested by Fig. 2. It will be seen, it is only somewhat above the

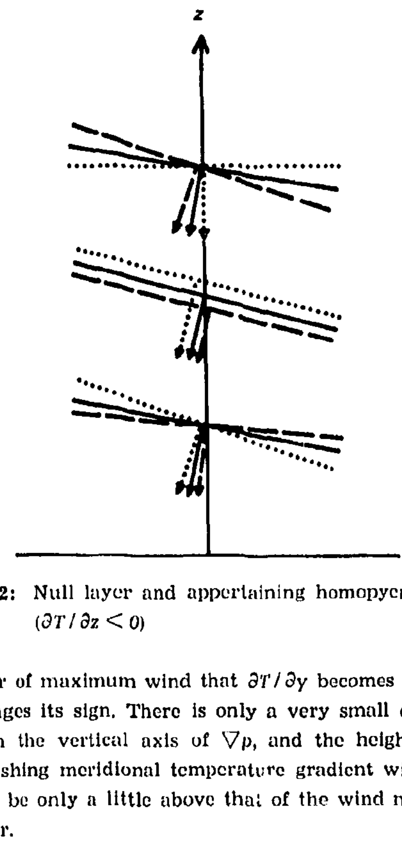


Fig. 2: Null layer and appertaining homopycnic layer ($\partial T/\partial z < 0$)

layer of maximum wind that $\partial T/\partial y$ becomes zero and changes its sign. There is only a very small deviation from the vertical axis of ∇p, and the height of the vanishing meridional temperature gradient will therefore be only a little above that of the wind maximum layer.

C. 5. **The layer of maximum winds where the large scale vertical motion becomes zero is the Null layer, found by H. Faust [7]. Slightly above the Null layer its corresponding layer of meridional temperature balance ($\partial T/\partial y = 0$; homothermic layer) is situated.**

When defining layers of vanishing meridional isobaric gradients that are analogous to the homothermic respectively homopycnic layer found in the z-system, it is seen that in the p-system, those are coincident with the Null layer which is already known to be barotropic. In other words, while the p-system shows only one layer, the

z-system has been found to include the following three layers, listed from high to low altitude:

Homothermic layer $\left(\dfrac{\partial T}{\partial y} = 0\right)$

Null layer $(\nabla T \times \nabla p = 0)$

Homopycnic layer resp. pleistobaric layer $\left(\dfrac{\partial \varrho}{\partial y} = 0;\ max\ of -\dfrac{\partial p}{\partial y}\right)$

It must be recalled that this is not a fact commonly noted in practical meteorological work, which is almost exclusively restricted to using the p-system.

A persistent wind maximum (Null layer) requires $\partial T/\partial y$ to change its sign closely above. From a theoretical point of view this may happen to occur on account of a layer originated from radiation and situated above the Null layer. Without investigating the physical possibilities of such a layer realizing, we see it is not necessary to introduce it in our model, as the adiabatic temperature changes that are due to the deduced vertical motion are considered to provide sufficient explanation.

The layer, $\partial T/\partial y = 0$, that the meridional temperature gradient changes its sign in, has been found to represent the first remarkable deviation from the working hypothesis (decreasing $\partial T/\partial y$ with altitude, see p. 5) adopted herein due to the basic suppositions. The working hypothesis will in the following be replaced by the deviation derived above.

*

If there is above the earth's surface warm air in areas of high pressure and cold air in those of low pressure $\left(\dfrac{\partial p_0}{\partial y} = 0\right)$, the differences in horizontal pressure grows with increasing height (until reaching the altitude of the homopycnic layer), and the resulting wind extreme is a west wind maximum. Above it, the isobaric temperature distribution must become inverse, since otherwise no wind extreme would come into existence.

C. 6 **Any persistent wind extreme layer will in the p-system be accompanied by a characteristic temperature distribution (quartered temperature field) which will be prevalent also in the z-system provided that the temperature changes caused by the vertical motion predominate over the decreasing temperature differences caused by the heating level.**

Theoretically, there may be a persistent wind extreme showing a quartered temperature field in the p-system (but not in the z-system); however, this is not very likely because the angular difference between **k** (vertical unit vector) and $-\nabla p$ is very small. Such case will not be considered in the following discussions.

It has been shown that it is only the vertical motion that accounts for the maintenance of such a temperature distribution. So, if there is a persistent wind maximum, high pressure must be connected with descending motion and low pressure with ascending motion underneath it. Continuity reasons lead to the conclusion that there is a flow of air masses from low to high pressure area in the layer of the persistent maximum wind. For its discrepancy against the equation of gradientic winds, (3), that flow of air masses is called 'nongradientic'.

C. 7 **In the Null layer, a nongradientic flow of air masses takes place from the high to the low latitudes (Null layer effect).**

Here again, it is important to note another fact. The vertical motion whose occurrence has been found to be necessary, will according to eq. (18) exist only on the assumption that $\partial T/\partial x$ and $\partial p/\partial x$ are not simultaneously equal to zero. That means, however, that the straightly uniform zonal conditions are departed from, as the x-direction is defined as being west-east. Thus, derivatives with respect to x show other values than zero only if the isolines of the respective meteorological parameters are not parallel to the latitudes (disturbances).

In setting up a model of the atmosphere this paper has so far been concerned only with mean conditions as used to describe the general circulation of the atmosphere. Disturbances of the general circulation, that is, the high-pressure and low-pressure areas, which are negligible in the average, have not been taken into account. However, as deduced by other authors earlier, it has been found here that the mean general circulation — although only a part of it has so far been deduced — cannot be maintained but by departure from the average conditions, that is to say, by the disturbances.

C. 8 **Disturbances of the mean general circulation are necessary for the maintenance of this mean circulation.**

*

The foregoing results have been based on the assumption that the pressure gradient is equal to zero on the earth's surface, other well-based assumptions not having been available. A modification will now be required:

C. 9 **The nongradientic flow of air masses from the low-pressure area (polar region) to the high-pressure area (low latitudes) that takes place at the height of the Null layer will cause air pressure on the earth's surface to drop in the polar region and to rise in the low latitudes. In the mean, west winds will consequently arise on the earth's surface.**

The flow of air masses taking place at the height of the Null layer from the polar region to the lower latitudes must be compensated at another height because other-

wise, the meridional pressure gradient would increase steadily. The discussion of this requires another basic fact to be employed which has not as yet been used, viz., the friction of the air on the earth's surface, which increases with the wind speed. This friction causes the actual wind speed to become subgradientic, that is, lower than the speed obtained from the gradientic wind equation (3). The stronger the friction (the higher the wind speed), the more subgradientic the wind will become.

The Coriolis force which under stationary, frictionless conditions balances the meridional pressure gradient, weakens on the existence of friction, so that the wind near the surface has a component toward low pressure (polar region).

C. 10 **Subgradientic winds will near the earth's surface cause a flow of air masses to take place from the low latitudes towards the polar region. This flow decreases with altitude.**

Air masses in the Null layer flowing toward high pressure, the wind in this layer, must be supergradientic, that is, its speed must be higher than would correspond to eq. (3) (gradientic wind equation). This flow of air masses toward high pressure decreases with increasing distance from the Null layer.

C. 11 **Between the earth's surface and the Null layer, there must be a layer where no flow of air masses takes place between high-pressure and low-pressure areas, i. e., where gradientic winds prevail (socalled level of non-divergence).**

It is also noteworthy that the opposite flows of air masses near the ground and in the Null layer are not always of the same value. First, there may be other nongradientic flows of air masses at higher altitudes and second, those may temporarily predominate over one another, causing the meridional pressure gradient to change. Such occurrence will not, however, prevail over a long period. If the predominant flow is that at the height of the Null layer (Null layer effect), stronger west winds will arise and owing to the heavier surface friction cause the flow near the ground towards the polar region to be stronger and of a balancing nature, and vice versa.

The Null layer effect causes the air masses to be transported to the low latitudes where the surface pressure must necessarily rise since it is representative of the weight of the whole of the air masses above the ground. In spite of the fact that due to surface friction the air masses flow towards the polar region, the surface pressure remains high. This is for the following reason.

A flow caused by surface friction and going towards low pressure, near the ground, cannot take place unless there is low pressure on the ground; pressure differences must have been caused by Null layer effect before the flow can start.

C. 12 **The Null layer effect maintains a surface pressure gradient which is directed from the low to the high latitudes ($\partial p_0 / \partial y < 0$).**

The foregoing statements that have been reached on the assumption that $\partial p_0 / \partial y = 0$ will in the below chapters be subjected to examination as to whether they will hold true also for $\partial p_0 / \partial y \neq 0$. We may anticipate that in the case of $\partial p_0 / \partial y < 0$ no changes will arise in respect of the conclusions relative to the Null layer.

The supposition that air density on the surface is higher in the polar region than in the low latitudes is based on the working hypothesis as to the nonexistence of a meridional pressure gradient on the earth's surface. But we have found that $\partial p_0 / \partial y$ is not equal to zero, and is negative. Both the left-hand side of eq. (7) and the first right-hand term of it are therefore negative (the latter in accordance with the basic facts). So, no conclusion is possible regarding the sign of $\partial \varrho_0 / \partial y$.

A proof of the existence of a positive, meridional density gradient on the earth's surface would thus be necessary (and in fact sufficient) to convey a proof of the conclusions made relative to the homopycnic layer.

In the low latitudes, the increase in pressure caused by the Null layer effect will in turn cause density to increase, and the increase in temperature caused by the descending vertical motion will cause density to decrease; analogous conditions apply to the polar region, vice versa. This does not prove whether it is the temperature or the pressure that has a predominant effect upon air density near the ground. The following consideration, too, cannot answer this question.

If we assume the effect of the temperature to predominate with respect to air density, ϱ_0 will be smaller in low latitudes than in high, which in turn would confirm what was said in the earlier deductions. But if we assume the pressure to have a greater effect on density than the temperature has, ϱ_0 will be larger in low latitudes than in high. No pleistobaric layer, and on the basis of the above conclusion, no Null layer would be provided. Further, $\partial p_0 / \partial y$ would result to be zero which would render higher values of ϱ_0 in the polar region (eq. (5)). The second assumption seems contradictory and only the first is left for operation.

There are, however, additional factors which have to be taken into account. It is not without the additional proof that no Null layer can exist without the homopycnic layer that the above consideration is correct, and to clarify this point, a number of other facts will have to be considered below.

Chapter 5: The Meridional Gradients of Air Pressure, Temperature and Density

Employing eq. (12)

(12)
$$\frac{\partial^2 p}{\partial z\,\partial y} = -g\frac{\partial}{\partial y}\left(\frac{p}{RT}\right) = -\frac{g}{RT}\left(\frac{\partial p}{\partial y} - \frac{p}{T}\frac{\partial T}{\partial y}\right)$$

we want to determine what is the condition of $\partial p/\partial y$ rising with height. In this event, the right-hand side of eq. (12) must be negative (since $\partial p/\partial y < 0$). The behavior, with height, of the meridional pressure gradient results to be

an increase / constant / a decrease if

(19) $$\frac{1}{p}\frac{\partial p}{\partial y} \gtreqless \frac{1}{T}\frac{\partial T}{\partial y}.$$

This nonequation will be of considerable importance in the following investigations. As yet no contradiction has arisen to it. Both the gradients are negative, that means, if $\partial p/\partial y$ is to increase with height, the relative gradient of temperature has to be larger absolutely than the relative gradient of pressure.

The denominator of eq. (3) (gradientic wind equation) includes the term ϱ. Since $1/\varrho$ increases with height, the wind speed must needs increase with height also in the event of the meridional pressure gradient keeping constant. It can be further concluded that there may even be a slightly decreasing meridional pressure gradient with height, while notwithstanding the wind speed increases with height. The point therefore is to which extent $-\partial p/\partial y$ has to decrease with height so as to preclude the occurrence of a wind maximum. This is the case if

(13) $$\frac{\partial v_x}{\partial z} = -\frac{1}{f\varrho}\left(\frac{\partial^2 p}{\partial z\,\partial y} - \frac{1}{\varrho}\frac{\partial p}{\partial y}\frac{\partial \varrho}{\partial z}\right)$$

remains negative, v_x, then, steadily decreasing with increasing height. The term enclosed by parentheses must therefore be positive, that is,

(20) $$\frac{\partial^2 p}{\partial z\,\partial y} > \frac{1}{\varrho}\frac{\partial p}{\partial y}\frac{\partial \varrho}{\partial z} = A$$

(Condition of wind speed decreasing with increasing height).

Both $\partial\varrho/\partial z$ and $\partial p/\partial y$ being negative renders nonequation (20) positive at the right hand, viz., $A > 0$. The opposite nonequality sign in nonequation (20) represents an increase of wind speed with height; and finally, in the case of $\partial^2 p/\partial z\,\partial y < 0$, which is identical with the upper nonequality sign in (19), even the meridional pressure gradient increases with height. When referring the foregoing considerations to each point of an air column, three possible cases are obtained, viz.,

I) $\dfrac{\partial^2 p}{\partial z\,\partial y} > A$: As long as this condition prevails, both wind speed and horizontal pressure gradient are decreasing with increase in height. Wind speed passes through zero (level of pressure equalization = homobaric layer) to increase in the opposite direction, which mathematically represents a change to negative values.

II) $A > \dfrac{\partial^2 p}{\partial z\,\partial y} > 0$: Wind speed increasing, whereas horizontal pressure gradient decreasing with height.

III) $\dfrac{\partial^2 p}{\partial z\,\partial y} < 0$: Both wind speed and horizontal pressure gradient increasing with height.

For each of these cases (for case I, however, only up to the level of pressure equalization), it is $\partial p/\partial y < 0$.

It has been shown that $\dfrac{\partial^2 p}{\partial z\,\partial y} = -g\dfrac{\partial \varrho}{\partial y}$

which renders $\partial\varrho/\partial y$ strongly negative for case I,
slightly negative for case II,
positive for case III.

In case I, the sign of $\partial\varrho/\partial y$ is equal to the sign of $\partial p/\partial y$. The required term is the sign of $\partial T/\partial y$. Introducing eq. (7) and (1) in nonequation (19) gives the following:

$$\frac{\partial p}{\partial z}\frac{\partial \varrho}{\partial y} > \frac{\partial p}{\partial y}$$

or

$$\frac{1}{RT}\frac{\partial p}{\partial z}\frac{\partial p}{\partial y} - \frac{\varrho}{T}\frac{\partial p}{\partial z}\frac{\partial T}{\partial y} > \frac{\partial p}{\partial y}\frac{\partial \varrho}{\partial z}$$

$$\frac{\dfrac{\partial p}{\partial y}\left(\dfrac{1}{R}\dfrac{\partial p}{\partial z} - T\dfrac{\partial \varrho}{\partial z}\right)}{\varrho\dfrac{\partial T}{\partial z}} > \varrho\frac{\partial p}{\partial z}\frac{\partial T}{\partial y}$$

This furnishes two solutions for $\partial T/\partial y$:

Ia) $\partial T/\partial y > 0$ (unlimited value)

Ib) $$0 > \frac{\partial T}{\partial y} > \frac{\dfrac{\partial T}{\partial z}}{\dfrac{\partial p}{\partial z}}\frac{\partial p}{\partial y}$$

Use of eq. (7) shows the following behavior of $\partial \varrho / \partial y$:

Case Ia: $$\frac{\partial \varrho}{\partial y} < \frac{\varrho}{p} \frac{\partial p}{\partial y} \; ; \quad \frac{1}{\varrho} \frac{\partial \varrho}{\partial y} < \frac{1}{p} \frac{\partial p}{\partial y}$$

Case Ib: $$\frac{\varrho}{p} \frac{\partial p}{\partial y} < \frac{\partial \varrho}{\partial y} < \frac{\frac{\partial \varrho}{\partial z}}{\frac{\partial p}{\partial z}} \frac{\partial p}{\partial y}$$

the right-hand term of the latter double nonequation representing the left-hand term of the nonequation of definition of case II. Hence, case I, where both wind speed and pressure gradient decrease with height, must be divided into two separate cases.

Case Ia: $$\operatorname{sign} \frac{\partial p}{\partial y} = \operatorname{sign} \frac{\partial \varrho}{\partial y} \neq \operatorname{sign} \frac{\partial T}{\partial y}$$

Case Ib: $$\operatorname{sign} \frac{\partial p}{\partial y} = \operatorname{sign} \frac{\partial \varrho}{\partial y} = \operatorname{sign} \frac{\partial T}{\partial y}$$

In case II, where increasing wind speed is accompanied by decreasing $-\partial p / \partial y$ with height, the following is obtained:

$$\frac{1}{\varrho} \frac{\partial p}{\partial y} \frac{\partial \varrho}{\partial z} > -g \frac{\partial \varrho}{\partial y} > 0$$

which by the use of eq. (7) yields

$$\frac{RT}{\varrho} \frac{\partial \varrho}{\partial z} \frac{\partial \varrho}{\partial y} + g \frac{\partial y}{\partial \varrho} + R \frac{\partial \varrho}{\partial z} \frac{\partial T}{\partial y} > 0 > g \frac{\partial \varrho}{\partial y}$$

Since $g \frac{\partial \varrho}{\partial y} < 0$ satisfies both cases I and II, the second nonequality sign can be ignored.

The factor by which $\partial \varrho / \partial y$ is to be multiplied is $g + \frac{RT}{\varrho} \frac{\partial \varrho}{\partial z}$ which can also be written $-R \frac{\partial T}{\partial z}$.

Hence,

$$-\frac{\partial \varrho}{\partial y} \frac{\partial T}{\partial z} + \frac{\partial \varrho}{\partial z} \frac{\partial T}{\partial y} > 0$$

The first three terms being negative, $\partial T / \partial y$ must be negative, too. Analogous to case Ib, the expression thus obtained for case II is:

$$\operatorname{sign} \frac{\partial p}{\partial y} = \operatorname{sign} \frac{\partial \varrho}{\partial y} = \operatorname{sign} \frac{\partial T}{\partial y}.$$

$\partial T / \partial y$ must, however, be stronger negative in case II than in case I, viz.,

$$\frac{\partial T}{\partial y} < \frac{\frac{\partial T}{\partial z}}{\frac{\partial \varrho}{\partial z}} \frac{\partial \varrho}{\partial y} \qquad \text{(left-hand limit; Fig. 3)}$$

This nonequation may be expressed by the parameters T and p (use of eq. (7)):

$$\frac{\partial T}{\partial y} < \frac{\frac{\partial T}{\partial z}}{\frac{\partial \varrho}{\partial z}} \frac{\partial \varrho}{\partial y} = \frac{\frac{1}{T} \frac{\partial T}{\partial z} \left(\frac{1}{R} \frac{\partial p}{\partial y} - \varrho \frac{\partial T}{\partial y} \right)}{\frac{1}{T} \left(\frac{1}{R} \frac{\partial p}{\partial z} - \varrho \frac{\partial T}{\partial z} \right)}$$

Then, after multiplication by the negative denominator and consequent reversion of the nonequality sign, the result is:

$$\frac{\partial T}{\partial y} \left(\frac{1}{R} \frac{\partial p}{\partial z} - \varrho \frac{\partial T}{\partial z} + \varrho \frac{\partial T}{\partial z} \right) > \frac{1}{R} \frac{\partial T}{\partial z} \frac{\partial p}{\partial y}$$

or

$$\frac{\partial T}{\partial y} < \frac{\partial p}{\partial y} \frac{\frac{\partial T}{\partial z}}{\frac{\partial p}{\partial z}}$$

The last nonequation and the last but two, which compares $\partial T / \partial y$ with $\partial \varrho / \partial y$, must not lead to the conclusion that their right-hand sides are mutually equal. It must not be disregarded that $\partial T / \partial y$ is implicit in both of these. If this were not so, a relation representing barotropic conditions would be obtained.

We must, however, now consider nonequation (19) which says that $\partial T / \partial y$ cannot be of unlimited value. Case II, where there still is a decreasing pressure gradient with height, shows the following:

$$\frac{\partial T}{\partial y} > \frac{T}{p} \frac{\partial p}{\partial y} \qquad \text{(right-hand limit; Fig. 3)}$$

which sets a limit for $\partial T / \partial y$ with respect to the other side. Hence, the limits for case II can be written as follows:

$$\frac{T}{p} \frac{\partial p}{\partial y} < \frac{\partial T}{\partial y} < \frac{\frac{\partial T}{\partial z}}{\frac{\partial p}{\partial z}} \frac{\partial p}{\partial y}$$

As for case III, where wind speed as well as pressure gradient are increasing with increasing height, $-g \frac{\partial \varrho}{\partial y} < 0$. $\partial p / \partial y$ being less than zero, $\partial T / \partial y$ must be negative due to eq. (7), and as a consequence, the condition in case III is that

$$\operatorname{sign} \frac{\partial p}{\partial y} = \operatorname{sign} \frac{\partial T}{\partial y} \neq \operatorname{sign} \frac{\partial \varrho}{\partial y}.$$

Eq. (7) requires $\partial T / \partial y$ to be stronger negative than $-\frac{T}{\varrho} \frac{\partial \varrho}{\partial y}$, which again, since $\partial p / \partial y$ must keep minus, yields nonequation (19).

$$\frac{\partial T}{\partial y} < \frac{T}{p} \frac{\partial p}{\partial y}$$

Hence, the strongest negative value of $\partial T / \partial y$ is reached in case III.

C. 13 **Investigation, in the z-system, of the vertical profiles of the meridional pressure gradient and of the scalar wind provides four substantially different sets of the meridional gradients of p, ϱ and T.**

For complement's sake another method is shown below by which to obtain the same results.

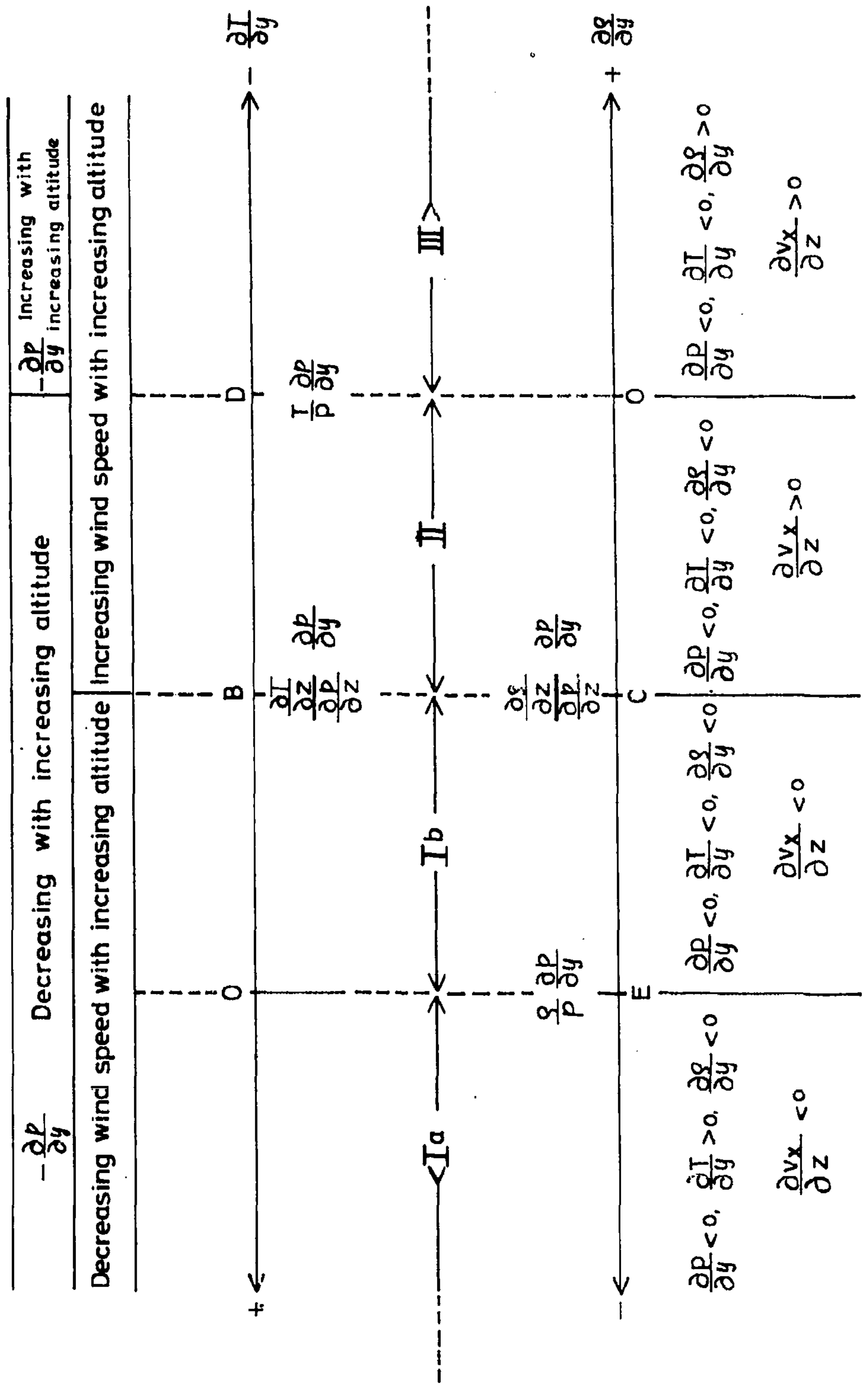


Fig. 3: The system of the sets for $\partial T/\partial z < 0$

The alternatives of the wind speed increasing and decreasing with height be defined by eq. (13a) and (15), wherein $\partial v_x / \partial z$ is expressed in the (T, p) and (ϱ, p) form respectively. This gives the same limits for $\partial T / \partial y$ and $\partial \varrho / \partial y$, as those deduced above. If in these equations $\partial T / \partial y$ and $\partial \varrho / \partial y$ are alternately made equal to zero, it appears that $\partial T / \partial y = 0$ can occur only in the event of decreasing wind speed with height, while $\partial \varrho / \partial y = 0$ can occur only in the event of increasing wind speed with height.

Below are presented the conditions necessary, and sufficient, for each of the four sets occurring. The relations of the signs as well as the nonequations having the absolute value signs, are independent of the direction of the y-axis. The respective nonequations shown beneath apply to neg. $\partial p / \partial y$. Fig. 3 presents the results in another form. The nonequation system (21—24) as well as Figure 3 hold for negative value of the vertical temperature gradient only. (Regarding positive vertical temperature gradient see page 38.)

Definition Equations for the four Sets ($\partial T / \partial z < 0$)

Set Ia

(21) $$\operatorname{sign} \frac{\partial p}{\partial y} = \operatorname{sign} \frac{\partial \varrho}{\partial y} \neq \operatorname{sign} \frac{\partial T}{\partial y}$$

$$\frac{\partial p}{\partial y} < 0; \quad \frac{\partial \varrho}{\partial y} < 0; \quad \frac{\partial T}{\partial y} > 0$$

Set Ib

(22) $$\operatorname{sign} \frac{\partial p}{\partial y} = \operatorname{sign} \frac{\partial \varrho}{\partial y} = \operatorname{sign} \frac{\partial T}{\partial y}$$

$$\left| \frac{\partial T}{\partial y} \right| < \left| \frac{\frac{\partial T}{\partial z}}{\frac{\partial p}{\partial z}} \frac{\partial p}{\partial y} \right|$$

$$\frac{\partial p}{\partial y} < 0; \quad \frac{\varrho}{p} \frac{\partial p}{\partial y} < \frac{\partial \varrho}{\partial y} < \frac{\frac{\partial \varrho}{\partial z}}{\frac{\partial p}{\partial z}} \frac{\partial p}{\partial y};$$

$$0 > \frac{\partial T}{\partial y} > \frac{\frac{\partial T}{\partial z}}{\frac{\partial p}{\partial z}} \frac{\partial p}{\partial y}$$

Set II

(23) $$\operatorname{sign} \frac{\partial p}{\partial y} = \operatorname{sign} \frac{\partial \varrho}{\partial y} = \operatorname{sign} \frac{\partial T}{\partial y}$$

$$\left| \frac{\frac{\partial T}{\partial z}}{\frac{\partial p}{\partial z}} \frac{\partial p}{\partial y} \right| < \left| \frac{\partial T}{\partial y} \right| < \frac{T}{p} \left| \frac{\partial p}{\partial y} \right|$$

$$\frac{\partial p}{\partial y} < 0; \quad \frac{\frac{\partial p}{\partial y}}{\frac{\partial p}{\partial z}} \frac{\partial \varrho}{\partial z} < \frac{\partial \varrho}{\partial y} < 0;$$

$$\frac{T}{p} \frac{\partial p}{\partial y} < \frac{\partial T}{\partial y} < \frac{\frac{\partial T}{\partial z}}{\frac{\partial p}{\partial z}} \frac{\partial p}{\partial y}$$

Set III

(24) $$\operatorname{sign} \frac{\partial p}{\partial y} = \operatorname{sign} \frac{\partial T}{\partial y} \neq \operatorname{sign} \frac{\partial \varrho}{\partial y}$$

$$\frac{\partial p}{\partial y} < 0; \quad \frac{\partial T}{\partial y} < 0; \quad \frac{\partial \varrho}{\partial y} > 0.$$

Chapter 6: The Meteorological Interpretation of the Results Deduced in Chapter 5

Any point in the atmosphere can be characterized by the sets being deduced in the previous chapter. For mean conditions in the atmosphere justly continual conditions can be assumed, thus each set has to show a definite vertical range of validity. Based on the physical connections between the particular meteorological field functions and their derivates a systematical vertical sequence of the set ranges is resulting, as it will be shown in chapter 12.

The set adjoining directly on the heating level (earth's surface) in case of undisturbed conditions and introducing this sequence will be called "bottom set" for the purpose of marking the condition of the atmosphere.

In set Ia the higher temperature is accompanied by low pressure, this is not consistent with our basic facts if this set adjoins on the earth's surface.

C. 14 **Bottom set Ia cannot be realized for mean conditions.**

Particular attention should, however be paid to sets Ib and II. They have no equivalent in the p-system which is almost exclusively used in meteorology. Deductions analogous to that of chapter 5 would only yield 2 sets in the p-system being formally equivalent to the sets Ia and III with meridional gradients formed within constant pressure levels. In this case density is a function of the temperature only and its gradient is contrarily to the meridional temperature gradient; therefore one distinctive mark vanishes. Consequently the sets Ib and Ia could not be separated unless by means of the geopotential Φ (= $g \cdot z$), sets II and III could not separated either.

C. 15 **Bottom set Ib represents a warm high pressure area and a cold low pressure area respectively, with wind speed decreasing with height. Consequently it cannot be realized for mean conditions.**

Bottom set Ib does not provide a Null layer and causes the pressure differences to equilize, thus leading again to the prerequisites to the development of sets II or III.

In respect of the sets Ib and II, where the meridional pressure gradient decreases with height, another essential conclusion may be drawn:

C. 16 **In the sets Ib and II the decrease in pressure with increasing altitude takes place at a faster rate in warm air than in cold air.**

This cannot be directly inferred from the p-system and therefore may appear strange. The decrease of pressure with height can be derived from eq. (1) (hydrostatic equation), and so is proportional to the air density: in the sets Ib and II, warm air shows higher density. Not being directly apparent in the p-system, it is not surprising, then, that these cases are not known from synoptical experience.

The above would show that merely the sets II and III on the earth's surface can be used for further study in establishing an atmospheric model of the extratropical latitudes, which at first is naturally restricted to average conditions. Later studies may show if the sets Ia and Ib are being important for unstationary conditions near the surface. Just the remaining sets leave open the problem arisen at the end of chapter 4: set II requires the meridional density gradient to be negative, while set III requires it to be positive. Set III has been used as a working hypothesis.

The difference between set II and set III is as follows:

$$(19) \qquad -\frac{1}{p}\frac{\partial p}{\partial y} \gtrless -\frac{1}{T}\frac{\partial T}{\partial y} \qquad \text{set}$$

$$\text{resp.}\ \frac{1}{p}\left|\frac{\partial p}{\partial y}\right| \gtrless \frac{1}{T}\left|\frac{\partial T}{\partial y}\right| \qquad \begin{matrix}\text{II}\\ \text{III}\end{matrix}$$

Both gradients being negative, superiority in respect of the air density gradient is exercised by the pressure gradient in set II, while by the temperature gradient in set III. In other words, if the value of the relative pressure gradient is higher than that of the relative temperature gradient, the pressure gradient and the density gradient will have the same direction (set II). It is not without the use of empirics that it can be clearly determined which of the relative gradients (between the pole and the low latitudes) has the higher value.

The basic facts that have been introduced in part I include also the velocity of the earth's rotation and the radiative conditions. These are now recalled. In 1947 *H. Faust* [8] showed — other theoretical publications on the subject by *H. Lettau* [9, 10] and *F. Wippermann* [11] followed — that with other conditions being equal, the differences in air pressure would be greater if the earth were rotating at a faster rate (increasing Coriolis force!). Analogously, the temperature differences between the high and the low latitudes would be greater, with other conditions being equal, if the earth's axis had a stronger inclination.

Since so far no result has been obtained that is contradictory to set II or set III, these must be considered to occur under particular circumstances. The mean condition depends upon the ratio between the probabilities of the occurrence of these two sets (for the distance pole to low latitudes), and this seems to be a function of the two abovementioned basic facts, i. e., the velocity of the earth's rotation and the slope of the earth's axis. The earth rotating at a faster rate would cause the probability of the occurrence of set II to rise, while a stronger slope of the earth's axis would cause the probability of the occurrence of set III to rise.

Chapter 7: The Homopycnic Layer in Bottom Sets II and III

No meteorological parameter can grow beyond all limits. It is only in the bottom sets II and III that the wind maximum that has been deduced in chapters 3 and 4 can originate. A pleistobaric layer, which according to eq. (13) must be underneath the layer of maximum winds (Null layer), can occur only in bottom set III. However, since there is a wind maximum in bottom set II as well, we again meet with the gap referred to above (p. 18).

C. 17 **A Null layer will necessarily occur wherever a pleistobaric layer exists (bottom set III); it can, however, occur also without a pleistobaric layer occurring (bottom set II).**

Both these bottom sets originating a Null layer, this has to be considered as being existent in the mean. This furnishes the outstanding proof (chapter 4) that a modification to the original assumption, $\partial p_0 / \partial y = 0$, will not affect the results attained in respect of the Null layer. According to eq. (15) and (18) the condition both necessary and sufficient of the existence of a Null layer has been offered by the parallelity of the three vectors, ∇p, ∇T, $\nabla \varrho$. $\partial p / \partial z$ and $\partial \varrho / \partial z$ keep negative in the whole of the atmosphere, and the meridional gradients, $\partial p / \partial y$ and $\partial \varrho / \partial y$, must therefore have the like sign within the Null layer.

It has been shown that sign $\partial p_0 / \partial y$ = sign $\partial \varrho_0 / \partial y$ in bottom set II. That means, since $\partial p / \partial y$ does not change its sign, $\partial \varrho / \partial y$, at least up to the height of the Null layer, will remain unchanged, too. From this is obvious again that in bottom set II there is no homopycnic layer beneath the Null layer. In bottom set III, however, understood owing to the fact that above the homopycnic layer, which represents the maximum value of the meridional pressure gradient, the latter decreases with altitude, although high pressure is connected with warm air and low pressure with cold air up to the height of the Null layer.

In other words, the difference between bottom sets II and III is that on the occurrence of the former, the homopycnic layer has sunk down to "below the earth's surface".

In the mean, which it is our primary aim to deduce here, the bottom sets II and III are existent side by side. While a Null layer is provided in both cases, and therefore exists in the mean, only one of these provides a homopycnic layer. So, may this be assumed to exist in the mean as well?

A collective system is assumed to be composed of portion A at a % and of portion B at b (= 100 - a) %. If each element of A possesses the same characteristic "C" (here, homopycnic layer), while each element of B lacks this characteristic, then the collective system possesses the characteristic "C" in the mean only provided that the portion B has not compensated it. For the mean of all bottom sets II and III a homopycnic layer may be considered to exist only if it proves to be causally necessary. That this necessity is not given can be seen from finding the mean of one each single bottom set II and III. When deriving the mean from the two curves $\partial \varrho / \partial y$ in Fig. 4, the resulting mean curve will no longer cross the zero line.

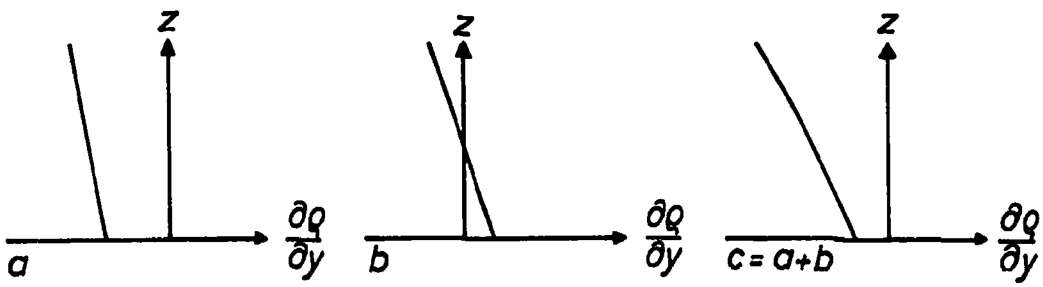


Fig. 4: Averaging two bottom sets

where sign $\partial p_0 / \partial y \neq$ sign $\partial \varrho_0 / \partial y$, $\partial \varrho / \partial y$ must change its sign (homopycnic layer) below the Null layer.

Thus, the validity of the equation of condition of set III expires in the homopycnic layer. Above this layer, the conditions in both bottom sets II and III are identical, such as, e. g., decrease of pressure with height being greater in warm air than in cold air. This is easily

Thus, for the mean of all bottom sets II and III the existence of the homopycnic layer does not prove causally necessary, and there still remains open the problem of the sign of the mean $\partial \varrho / \partial y$.

In the following investigations the two bottom sets, II and III, will therefore have to be ascribed equal rights to.

Chapter 8: The Layers of Maximum Meridional Gradients of Air Temperature and Air Density above the Null Layer

Other modifications to the basic assumptions have meanwhile proved necessary. In bottom set III, $\partial\varrho/\partial y$ does not move asymptotically toward zero with increasing height but reaches zero already in H_ϱ, becoming negative above same. Nor is the basic assumption met by the connected $\partial p/\partial y$, which shows an extreme value at the height of H_ϱ.

The negative value of the meridional density gradient cannot grow beyond all limits above the homopycnic layer. Within the theoretical equalization layer, at a very high altitude — it is not a matter for discussion here if same is realized in nature —, $\partial\varrho/\partial y$ must be equal to zero. At some height above H_ϱ $\partial\varrho/\partial y$ must therefore reach a (negative) maximum value.

Bottom set II has shown $\partial\varrho/\partial y$ to keep negative below the Null layer. At first an asymptotic approach of it to zero seems possible and for this reason, we shall ask for the general conditions of the occurrence of a pleistopycnic layer.

We shall employ the relations connected with a wind extreme layer, and our investigations will again be restricted to the zonal component of the wind. In order to find out the relative position of the Null layer and the pleistopycnic layer, the behavior of $\partial^2\varrho/\partial z\,\partial y$ at the height of the Null layer will be considered.

Another derivative of eq. (13) with respect to z is formed:

$$\frac{\partial^2 v_x}{\partial z^2} = \frac{g}{f\varrho^2}\frac{\partial\varrho}{\partial z}\frac{\partial\varrho}{\partial y} + \frac{g}{f\varrho}\frac{\partial^2\varrho}{\partial y\,\partial z} + \frac{1}{\varrho^2}\left(\frac{\partial\varrho}{\partial z}\right)^2 v_x - \frac{1}{\varrho}\frac{\partial^2\varrho}{\partial z^2}v_x - \frac{1}{\varrho}\frac{\partial\varrho}{\partial z}\frac{\partial v_x}{\partial z} \tag{25}$$

The condition of the occurrence of a Null layer renders the last member of this equation zero. For the west wind maximum it is $\partial^2 v_x/\partial z^2 < 0$, which provides the following nonequation

$$\frac{\partial^2\varrho}{\partial y\,\partial z} < \frac{1}{\varrho}\frac{\partial\varrho}{\partial z}\frac{\partial\varrho}{\partial y} - \frac{f}{g\varrho}\left(\frac{\partial\varrho}{\partial z}\right)^2 v_x + \frac{f}{g}\frac{\partial^2\varrho}{\partial z^2}v_x \tag{26}$$

(for the layer where $\partial v_x/\partial z = 0$)

Considering the Null layer conditions, this can be further simplified. Eq. (1) and (3) render

$$v_x = \frac{g}{f}\frac{\dfrac{\partial p}{\partial y}}{\dfrac{\partial p}{\partial z}} \tag{27}$$

Here it must be noted that a Null layer is connected with the parallelity of the layers of constant meteorological parameters (cf. chapter relative to Null layer). For (y, z)-dependence, this parallelity is designated by

$$-\frac{\dfrac{\partial\alpha}{\partial y}}{\dfrac{\partial\alpha}{\partial z}} = -\frac{\dfrac{\partial\beta}{\partial y}}{\dfrac{\partial\beta}{\partial z}} \tag{28}$$

where α, β denote two parameters.

Applying the above to p and ϱ, nonequation (26) reads as follows:

$$\frac{\partial^2\varrho}{\partial y\,\partial z} < \frac{1}{\varrho}\frac{\partial\varrho}{\partial z}\frac{\partial\varrho}{\partial y} - \frac{1}{\varrho}\left(\frac{\partial\varrho}{\partial z}\right)^2\frac{\dfrac{\partial p}{\partial y}}{\dfrac{\partial p}{\partial z}} + \frac{fv_x}{g}\frac{\partial^2\varrho}{\partial z^2}$$

where the first two terms at the right hand can be transformed to

$$\frac{1}{\varrho}\left(\frac{\partial\varrho}{\partial z}\right)^2\left\{\frac{\dfrac{\partial\varrho}{\partial y}}{\dfrac{\partial\varrho}{\partial z}} - \frac{\dfrac{\partial p}{\partial y}}{\dfrac{\partial p}{\partial z}}\right\}$$

In the case of a wind maximum the term enclosed by brackets becomes zero, which leaves

$$\frac{\partial^2\varrho}{\partial y\,\partial z} < \frac{fv_x}{g}\frac{\partial^2\varrho}{\partial z^2} \quad \left(\text{for } \frac{\partial v_x}{\partial z} = 0\right). \tag{29}$$

$\partial\varrho/\partial z$ is negative and in the mean, its value decreases with height. Therefore, $\partial^2\varrho/\partial z^2 > 0$, and in this case, $v_x > 0$ applies also. For a simplification $\dfrac{f}{g}v_x\dfrac{\partial^2\varrho}{\partial z^2} = \psi^2$ is introduced and the expression now reads:

$$\frac{\partial^2\varrho}{\partial y\,\partial z} < \psi^2 \tag{29 a}$$

Hence, the layer of maximum (negative) $\partial\varrho/\partial y$ may be met with already below the Null layer. But since an unlimited number of negative values, while only a limited of positive values (0 to ψ^2) are available for $\partial^2\varrho/\partial z\,\partial y$, the required pleistopycnic layer in nature probably is more frequently situated above the Null layer rather than below.

The above does not offer a satisfactory proof and another criterion will have to be sought. For Null layer condi-

tions (where $\partial v_x / \partial z = 0$, among others), the following is obtained by use of eq. (1), (3) and (7):

$$\frac{\partial \varrho}{\partial y} = -\frac{\varrho}{T}\left(\frac{f}{R} v_x + \frac{\partial T}{\partial y}\right)$$

$$\frac{\partial}{\partial z}\left(\frac{\partial \varrho}{\partial y}\right) = \left(-\frac{1}{T}\frac{\partial \varrho}{\partial z} + \frac{\varrho}{T^2}\frac{\partial T}{\partial z}\right) \cdot \left(\frac{f}{R} v_x + \frac{\partial T}{\partial y}\right) - \frac{\varrho}{T}\frac{\partial^2 T}{\partial y \partial z}$$

$$\frac{\partial}{\partial z}\left(\frac{\partial \varrho}{\partial y}\right) = -\frac{\varrho}{T}\left\{\frac{\partial^2 T}{\partial y \partial z} + \frac{1}{\varrho}\frac{\partial \varrho}{\partial y}\left(\frac{g}{R} + 2\frac{\partial T}{\partial z}\right)\right\} \tag{30}$$

The position of the pleistopycnic layer relative to that of the Null layer is found from nonequation

$$\frac{1}{\varrho}\frac{\partial \varrho}{\partial y}\left(\frac{g}{R} + 2\frac{\partial T}{\partial z}\right) + \frac{\partial^2 T}{\partial y \partial z} \gtreqless 0 \quad \text{pleistopycnic layer} \left\{\begin{matrix}\text{above}\\ \text{within}\\ \text{below}\end{matrix}\right\} \text{Null layer} \tag{30 a}$$

The first member is negative, the second positive. The latter is supposed to be the predominant one, as already pointed out by W. Attmannspacher in [5].

Nonequation (30 a) does not satisfactorily determine the position of the pleistopycnic layer, either.

It is only from bottom set III that the existence of this layer has cogently resulted. In the vicinity of the Null layer, $\partial \varrho / \partial y$ has the like sign in bottom sets III and II, as would appear also from eq. (31).

$$\frac{\partial \varrho}{\partial y} = \frac{f}{g}\frac{\partial \varrho}{\partial z} v_x \qquad \left(\text{for } \frac{\partial v_x}{\partial z} = 0\right) \tag{31}$$

Although bottom set II does not show whether or not a pleistopycnic layer develops, the effects of bottom set III cannot be compensated in the mean for the above-mentioned reason, and contrary to what was said in connection with Fig. 4.:

C. 18 **In the vicinity of the Null layer, and probably mostly above it, there is a pleistopycnic layer.**

*

Somewhat over the Null layer the meridional temperature gradient changes its sign and becomes positive, this applying to both bottom sets II and III. Using the axiom that no meteorological parameter can grow beyond all limits, the following can be stated: —

C. 19 **Above the Null layer, there must develop a layer of maximum meridional temperature gradient (pleistothermic layer).**

It will now have to be ascertained whether the pleistothermic layer is above the deduced pleistopycnic layer or whether it is below.

Forming the derivative of eq. (7) with respect to z renders the following for the layers of maximum meridional gradients:

$$\frac{\partial^2 \varrho}{\partial z \partial y} = \frac{1}{T}\left(\frac{1}{R}\frac{\partial^2 p}{\partial z \partial y} - \frac{\partial T}{\partial z}\frac{\partial \varrho}{\partial y} - \frac{\partial \varrho}{\partial z}\frac{\partial T}{\partial y} - \varrho\frac{\partial^2 T}{\partial z \partial y}\right) = 0 \tag{32 a}$$

$$\frac{\partial^2 T}{\partial z \partial y} = \frac{1}{\varrho}\left(\frac{1}{R}\frac{\partial^2 p}{\partial z \partial y} - \frac{\partial T}{\partial z}\frac{\partial \varrho}{\partial y} - \frac{\partial \varrho}{\partial z}\frac{\partial T}{\partial y} - T\frac{\partial^2 \varrho}{\partial z \partial y}\right) = 0 \tag{32 b}$$

In the pleistothermic layer $\partial^2 T / \partial z \partial y = 0$, hence

$$\frac{\partial^2 \varrho}{\partial z \partial y} = \frac{1}{T}\left(\frac{1}{R}\frac{\partial^2 p}{\partial z \partial y} - \frac{\partial T}{\partial z}\frac{\partial \varrho}{\partial y} - \frac{\partial \varrho}{\partial z}\frac{\partial T}{\partial y}\right)$$

where the first term at the right hand can by use of the hydrostatic equation be written as $-\frac{g}{R}\frac{\partial \varrho}{\partial y}$; then the first two terms render jointly

$$-\left(\frac{g}{R} + \frac{\partial T}{\partial z}\right)\frac{\partial \varrho}{\partial y}$$

Hence, the below relationship holds in the pleistothermic layer:

$$\frac{\partial^2 \varrho}{\partial z \partial y} = \frac{1}{T}\left\{-\left(\frac{g}{R} + \frac{\partial T}{\partial z}\right)\frac{\partial \varrho}{\partial y} - \frac{\partial \varrho}{\partial z}\frac{\partial T}{\partial y}\right\}$$

The positive g/R is the equivalent of the negative value of the vertical temperature gradient of the homogeneous atmosphere; $\partial T / \partial z$ which occurs in the real atmosphere, and which is negative if the earth's surface is assumed to be the only heating level, is considerably smaller.

The member in parentheses at the right hand is positive. $\partial \varrho / \partial y$ being negative above $H\varrho$ in both bottom sets II and III, the first right-hand term, including the sign, is positive. $\partial \varrho / \partial z$ is always negative, while $\partial T / \partial y$ is positive above the homothermic layer. The second ter... at the right hand, including the sign, therefore is positive too.

The latter equation applies only to the pleistothermic layer. So $\partial^2 \varrho / \partial z \partial y$ is positive there, and since $\partial \varrho / \partial y$ is negative, it must decrease with altitude in the vicinity of the pleistothermic layer.

C. 20 **The layer of maximum meridional (negative) density gradient (pleistopycnic layer) is situated below the pleistothermic layer.**

The abovededuced results show that with increasing altitude, the layers of zero (bottom set III only) and of the maximum of the meridional gradient are reached first by the air density, then by the air temperature.

Chapter 9: The Influence of the Vertical Motion on the Temperature and Density Fields

The vertical motion that is connected with the Null layer has, in turn, an influence on the fields of the meteorological parameters. In Fig. 5 the solid lines illustrate the "undisturbed" temperature height curves for high and low latitudes below the Null layer. By warming due to descending air in the low latitudes, and cooling due to ascending air in the high latitudes, these curves are modified in such a way as represented by the dashed lines. On the earth's surface and within the Null layer no change of temperature takes place. The original tem-

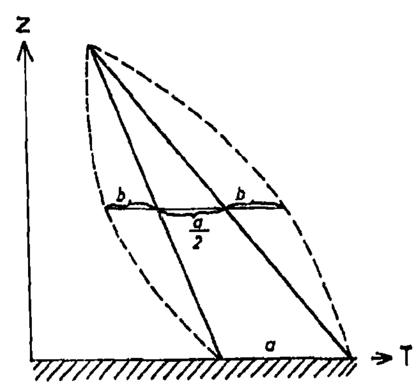


Fig. 5: The influence of the large-scale vertical motion on the meridional temperature gradient

perature differences at half the altitude between earth's surface and Null layer be denoted by $a/2$, while the temperature differences due to vertical motion be denoted by 'b'; then, a layer of maximum meridional gradient will originate if $2b > a/2$. Because the curves representing the original temperature conditions diverge as they come to the earth's surface, the developing pleistothermic layer will be situated somewhat below the layer of maximum vertical motion.

C. 21 **The pleistothermic layer that is originated below the Null layer by vertical motion, is situated underneath the layer of maximum vertical motion.**

Validity of the foregoing result is not restricted to the conditions prevailing between the high and the low latitudes. It has been shown earlier that there is a necessity for the meteorological parameters to deviate from their latitudinal parallelity (high-pressure and low-pressure systems). The result also holds true for the vertical motion within the disturbances, and is even of primary importance for these. A pleistothermic layer that is situated below the Null layer has been found to exist only if $2b > a/2$, and in view of the great temperature difference 'a' between the high and the low latitudes it is unlikely that the vertical motion originates a pleistothermic layer that is situated between the Null layer and the earth's surface. However, the "undisturbed" temperature height curves along one latitude coincide, and in the event of a high-pressure area and a low-pressure area developing on the same latitude, a marked pleistothermic layer will develop below the Null layer. Due to additional, radiative warming within the high, 'a' will be different from zero and the pleistothermic layer will be situated slightly below the layer of maximum vertical motion.

The deduced necessity of the existence of a pleistothermic layer below the layer of maximum vertical motion applies to the sets II and III equally. These two sets on the ground must, however, be separated again when the problem of the existence of a pleistopycnic layer, connected with the pleistothermic layer, is being investigated. While the temperature gradient normal to the flow is negative (towards the high) in both cases, the respective density gradient is negative in set II, and positive in set III.

Set III will be dealt with first.

The condition in the pleistothermic layer below H_Q is as follows:

$$\text{(32b*)}\quad \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial n}\right) = \frac{1}{R}\frac{\partial}{\partial z}\left(\frac{1}{\varrho}\frac{\partial p}{\partial n}\right) - \frac{\partial}{\partial z}\left(\frac{T}{\varrho}\frac{\partial \varrho}{\partial n}\right) = 0$$

$\partial p/\partial n$ has been defined as being negative, this feature increasing with height. The positive $1/\varrho$ increases with height, which renders the first term at the right-hand side of eq. (32b*) negative, and in order that this equation may be equal to zero, the second term at the right hand has to be positive. Differentiating, and considering the sign, renders

$$-\frac{T}{\varrho}\frac{\partial}{\partial z}\left(\frac{\partial \varrho}{\partial n}\right) - \frac{\partial \varrho}{\partial n}\frac{\partial}{\partial z}\left(\frac{T}{\varrho}\right) > 0$$

T/ϱ is positive and increases with height, and for this reason, $\partial/\partial z\,(T/\varrho)$ is positive too. The nonequation will not be satisfied unless $\partial/\partial z\,(\partial\varrho/\partial n)$ is greater than zero. Since $\partial\varrho/\partial n$ decreases with height in the pleistothermic layer, the pleistopycnic layer cannot be above the same. The following applies to the pleistopycnic layer by analogy with the above:

$$(32a^*) \quad \frac{\partial}{\partial z}\left(\frac{\partial \varrho}{\partial n}\right) = \frac{1}{R}\,\frac{\partial}{\partial z}\left(\frac{1}{T}\,\frac{\partial p}{\partial n}\right) - \frac{\partial}{\partial z}\left(\frac{\varrho}{T}\,\frac{\partial T}{\partial n}\right) = 0$$

$\frac{1}{T}\frac{\partial p}{\partial n}$ is negative, and its negative value increases with height. The first term at the right hand is negative, and to satisfy eq. 32a*, the second term has to be positive. Differentiating the second term renders

$$-\frac{\varrho}{T}\,\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial n}\right) - \frac{\partial T}{\partial n}\,\frac{\partial}{\partial z}\left(\frac{\varrho}{T}\right) > 0$$

According to definition, $\partial T/\partial n$ is negative; ϱ/T is positive and decreases with altitude. Hence, $\partial/\partial z\,(\varrho/T)$ is negative. The nonequation is not satisfied unless $\partial/\partial z\ (\partial T/\partial n)$ is negative; that means, the negative $\partial T/\partial n$ must increase with height in the pleistopycnic layer.

C. 22 **The layer of maximum density gradient normal to the flow is situated slightly underneath the layer of maximum temperature gradient normal to the flow (bottom set III).**

In set III the signs of $\partial T/\partial n$ and $\partial \varrho/\partial n$ are opposite below $H\varrho$. With $\partial p/\partial n$ assumed to keep constant, an increase of the negative temperature gradient must according to eq. (7) cause an increase of the positive density gradient, and vice versa.

In set II both the gradients dealt with show the same sign. So, with $\partial p/\partial n$ assumed to be constant, intensification of one gradient normal to the flow would have to cause weakening of the other, and vice versa. However, since we cannot say if $\partial p/\partial n$ will indeed keep constant, this is not a cogent result. It would mean that below the layer of maximum temperature gradient there would be a layer of minimum density gradient, which latter it is for the first time that we have to consider.

With respect to the mean course of the meridional density gradient in the vertical range considered no final statement can be made because neither the course and value of $\partial \varrho/\partial y$ nor the relative frequency of both bottom sets are definitely known. Mean conditions are possible with failing extreme, with a (positive) maximum or a (negative) minimum of the meridional density gradient.

Chapter 10: The Influence of the Inhomogeneous Surface of the Earth

The earth's surface has so far been mainly regarded as a heating level to produce a meridional temperature gradient. To be able to see what an influence is exerted on the surface pressure gradient by the earth's surface as an impenetrable level (ignoring surface friction for the time being), it will be regarded as a penetrable level similar to differently heated wire netting.

A later part (III) of this analysis will show that at a specific altitude, below some heating level in the free atmosphere, a layer originates in which the meridional pressure gradient becomes zero (homobaric layer). Thus, through the earth's surface as a rigid level, a thermal, static pressure equilibrium cannot be reached — the meridional pressure gradient remains different from zero.

The results deduced in respect of the dynamics of the planetary circulation hold true analogously for the system of high-pressure area — low-pressure area. This analogy may, however, be deviated from due to the influence exerted by the inhomogeneous surface of the earth; in the planetary system the horizontal temperature gradient near the surface keeps its direction in the mean, whereas it may show any direction between high-pressure area and low-pressure area, due to the influence of the inhomogeneous earth surface. Such an influence is represented by the change of the ground underneath wandering pressure systems.

If near the surface the temperature arrow is positioned between the pressure arrow and the z-axis, the two arrows as they move upward will be parallel to each other at a certain position. In such cases, at this specific altitude which chiefly is dependent upon the respective temperature differences at the earth's surface, a layer of extreme winds can develop. This can only be a layer of minimum winds, for this extreme is below a wind maximum and there is no other wind extreme between them (parallelity of arrows p and T on Fig. 6).

It will be shown in later chapters that a layer of persistent wind minimum too, is a Null layer, that is, a layer with reversing vertical motion. In the case dealt with here, there is a quartered temperature field in the p-system; but there is none in the z-system because of the fact that the temperature arrow in Figs. 6a and 6b keeps always to the left. This, for example, may be the case in winter. The criterion regarding vertical motion used here has been deduced for adiabatic conditions, which do not apply to the case under reference. Therefore, the vertical motion will not be discussed here. Notice only the following statement:

C. 23 **Within the system of high-pressure area — low-pressure area, a wind minimum often develops in the lower part of the atmosphere, if there is a relatively small temperature gradient near the earth's surface. This minimum must be situated underneath of the pleistothermic layer.**

The behavior of $\partial\varrho/\partial n$ has not been evaluated yet (the positive n-axis at the lower boundary of the undisturbed range is directed from the center of the high to the center of the low). When taking up this problem we are led to the set characteristics defined in chapter 5. Fig. 6 offers a view of all possible combinations that are in accordance with the conditions of the disturbances, the dashed line showing the upper boundary of the disturbed range. The foregoing considerations hold for a weakened $\partial T/\partial n$, which however does not change its sign.

Considerations of the behavior of $\partial\varrho/\partial n$ above the disturbed range makes obvious a difference within the disturbed range itself (Figs. 6a, 6b).

C. 24 **In the surface-disturbed system of high-pressure area — low-pressure area with $(\partial T/\partial n)_o < 0$, there are between the developed wind minimum and the Null layer either two layers of vanishing $\partial\varrho/\partial n$, one of these being within the disturbed range, or no such layer.**

In the mean planetary system, a layer where $\partial\varrho/\partial n = 0$ corresponds to a homopycnic layer and therefore will in the following be termed „analogical homopycnic layer"; accordingly, all the other layers of the y-system will in the n-system be given the prefix "analogical". For the sake of clear arrangement, the analogical pleistothermic and analogical pleistopycnic layers have not been plotted in Fig. 6.

Figs. 6c and 6d indicate the situation prevailing in the event of $\partial T/\partial n$ even changing its sign due to particularly strong surface disturbances.

C. 25 **In the surface-disturbed system of high — low, with $(\partial T/\partial n)_o > 0$, there are between the developed wind minimum and the Null layer either two analogical homopycnic layers, one of these being within the disturbance range, or none. Additionally, an analogical homothermic layer is present below the wind minimum in each such case.**

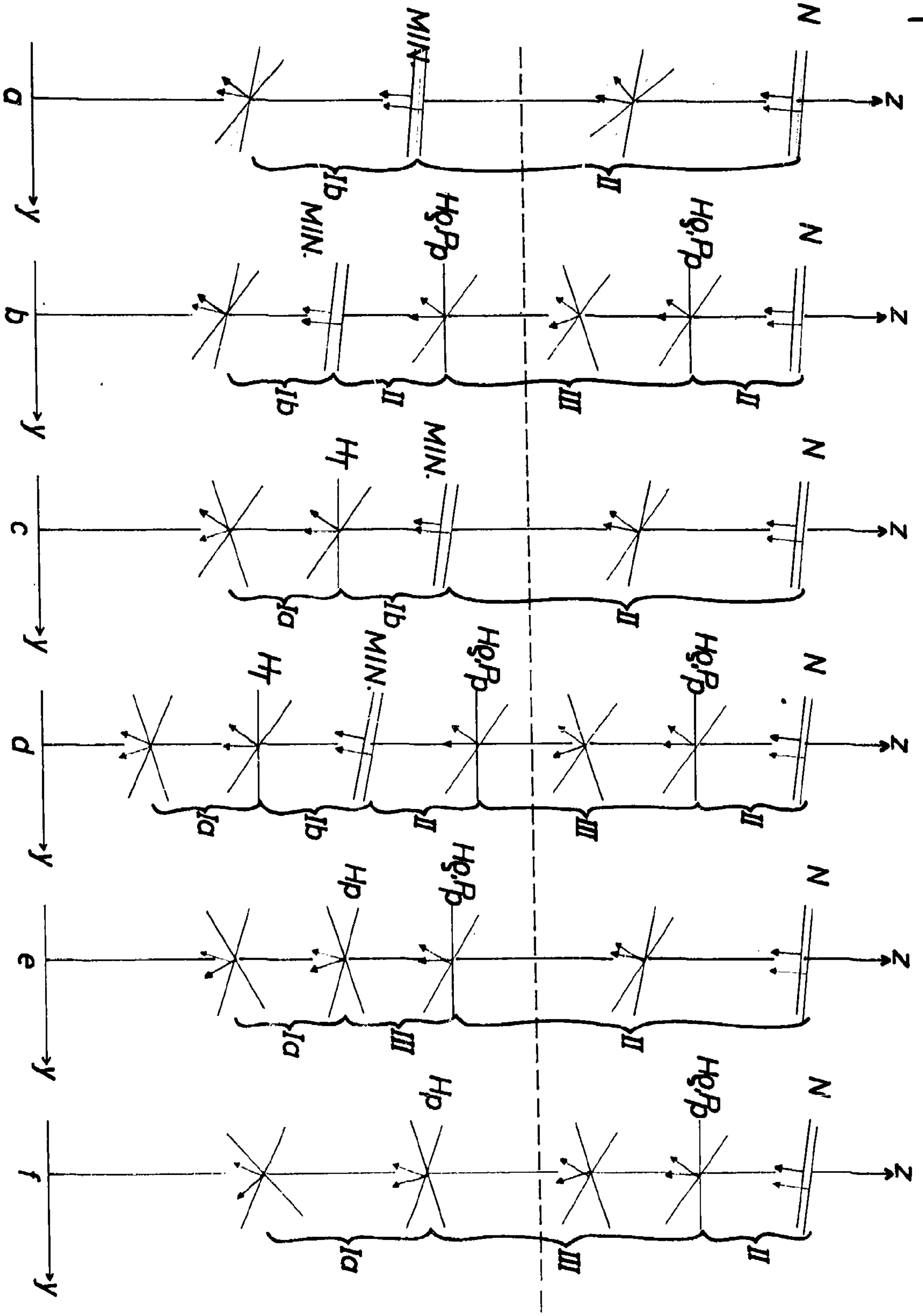


Fig. 6: Disturbed bottom sets

It has so far been assumed that the surface disturbance causes $(\partial T / \partial n)_o$ either to decrease (Figs. 6a, b) or to change its sign (Figs. 6c, d). Let us now consider the possibility of the originally existing $\partial T / \partial n$ being intensified by surface disturbances. Its sufficient intensification will cause the character of the pressure systems of the disturbance range to change.

Static effects obviously cause dynamic processes in this case. The descending, very cold air causes a horizontal convergence of air masses in its upper sphere while the ascending, very warm air causes a horizontal divergence of air masses. These two processes may occur jointly or individually; in either case an individudal circulation develops within the disturbance range. The mass flow resulting from the aforementioned divergence or convergence must be considered the cause of the reversal of air pressure differences below the analogical homobaric layer.

C. 26 **In the surface-disturbed system of high — low, with $(\partial T / \partial n)_o < 0$ and $(\partial p / \partial n)_o > 0$, an analogical homobaric layer develops within the disturbance range. Between this layer and the Null layer there is an analogical homopycnic layer either within the disturbance range or above it.**

Since the earth's surface in reality is a rigid and rough medium, the wind speed is considerably decreased in its vicinity.

C. 27 **If the deduced wind minimum is situated above the zone affected by surface friction, a wind maximum develops below it.**

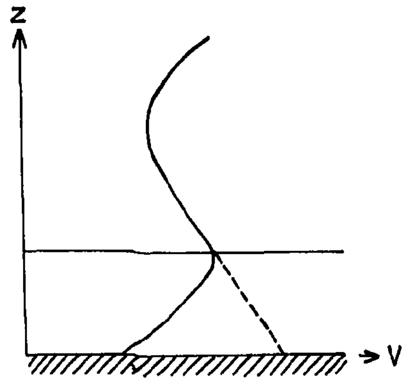


Fig. 7: Cut-off maximum

Formation of this wind maximum as a cut-off maximum is illustrated in Fig. 7. Having been caused by an additional force, it cannot be found from evaluating the three gradients.

Chapter 11: Details of the Horizontal Gradients

When replacing in eq. (18) the (p, T)-form of the vector product by the two other forms from eq. (14), the following is obtained:

(18b) $$v_z = \frac{RT}{f\varrho(\gamma + \Gamma)} (\nabla T \times \nabla\varrho)_{z\text{-comp.}} = \frac{1}{f\varrho^2(\gamma + \Gamma)} (\nabla p \times \nabla\varrho)_{z\text{-comp.}}$$

From a purely mathematical point of view this means that the vertical motion becomes zero also in the level of $\nabla_h\varrho = 0$. But as the homopycnic layer does not represent a zero level of the vertical motion, a comparison will be made between the two statements so as to avoid thinking difficulties.

The turn to zero and reversal of the vertical motion within the Null layer is effected by the parallelity of the vectors, ∇p, ∇T and $\nabla\varrho$; not to the zero condition of any of the vectorial factors of (18) or (18b). Up to this we have not found a layer where any of the three horizontal gradients may vanish in a particular case. In the system of planetary circulation the homopycnic layer has been defined as $\partial\varrho/\partial y = 0$. $\partial\varrho/\partial x$ as well becomes zero in the mean, and it is only in the mean that $H\varrho$ indeed represents a layer with $\nabla_h\varrho = 0$. However, if the mean values of the horizontal gradients were placed in the equations for v_z, i. e. if all x-components were rendered equal to zero, the vertical motion would be zero in all layers. In spite of the fact that the x-components compensate one another in the mean, the vertical motions do not; this is for the reason that in the component form of the vector products any derivative with respect to x possesses a weight factor showing a derivative with respect to y.

The product of two mean values is equal to the mean of the individual products only where there is no correlation between the factors. The correlation that exists in the equations for v_z will be commented upon in a later analysis.

We again are confronted with what might appear to be strange — prerequisite to the maintenance of the mean conditions are deviations from them (see p. 17). The component form of eq. (18b) reads as follows:

(18b) $$v_z = \frac{RT\left(\dfrac{\partial T}{\partial x}\dfrac{\partial\varrho}{\partial y} - \dfrac{\partial T}{\partial y}\dfrac{\partial\varrho}{\partial x}\right)}{f\varrho\left(\dfrac{\partial T}{\partial z} + \Gamma\right)} = \frac{\left(\dfrac{\partial p}{\partial x}\dfrac{\partial\varrho}{\partial y} - \dfrac{\partial p}{\partial y}\dfrac{\partial\varrho}{\partial x}\right)}{f\varrho^2\left(\dfrac{\partial T}{\partial z} + \Gamma\right)}$$

A reversal of the large-scale vertical motion has so far been observed only vertically; there must be areas of opposite vertical motion horizontally also. The geometric locus of all separating lines that belong together would be a generally curved sloping surface. These surfaces, in conjunction with the Null layer, will separate areas of like-signed vertical motion from one another.

According to definition, $\partial\varrho/\partial y = 0$ in the homopycnic layer, whereas both $\partial T/\partial y \neq 0$ and $\partial p/\partial y \neq 0$ (see chapter 5). In this layer, v_z disappears only at the lines of its intersection with the above-described surfaces. Eq. (18b) requires $\partial\varrho/\partial x$ to be zero in these intersections. The singular places in the homopycnic layer at which in fact $\nabla_h\varrho = 0$ are represented by lines that run between the areas of ascending and descending air.

It is for the first time in this study that we meet with a meteorologic characteristic, $\nabla_h\varrho = 0$, expressing itself only in a one-dimensional form, a line. Mathematically, this is however easily understood. $\nabla_h\varrho = 0$ includes two mutually independent conditions, $\partial\varrho/\partial x = 0$ and $\partial\varrho/\partial y = 0$. That geometric form in the three-dimensional space which must satisfy only one condition has two degrees of freedom; it is a surface. Thus, two mutually independent conditions can be simultaneously satisfied only by a line, which is imagined as the intersection of two surfaces. This, of course, disregards the contingency of the two conditional surfaces coinciding.

Notice also that the above results hold true analogously for the homothermic layer. The conditions in the vicinity of the homobaric layer will be interpreted in the next chapter.

Chapter 12: The Conditions at Higher Altitudes

A pleistopycnic layer has been found to exist above the Null layer. To evaluate the conditions at still higher altitudes, the system of high and low latitudes will be re-adopted, i. e. the disturbances will be neglected for the time being. Above the pleistopycnic layer the meridional density gradient must necessarily decrease. At these altitudes the density in the low latitudes is greater than in the high, and as long as this condition remains, pressure in the low latitudes must decrease with height at a faster rate than in the high latitudes, due to the hydrostatic equation.

homobaric layer the disturbances will again be taken into account. The component form of eq. (18a) indicates that the vertical motion does not become zero in this homobaric layer ($\partial p / \partial y = 0$). The idea suggests itself that $\partial p / \partial x$, too, becomes zero therein, though there has not been found yet a layer where the two gradientic components become zero simultaneously. Our assumption is that the vertical motion guided by the Null layer compensates, and finally overcompensates the slope of the pressure level, which too is influenced by disturbances. It has been shown in chapter 4 that barotropic

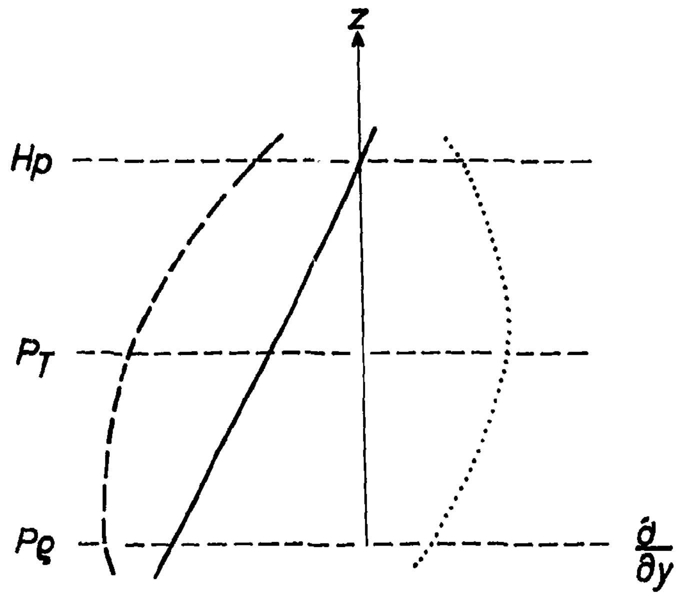


Fig. 8: The vicinity of the homobaric layer

This finally makes the meridional pressure gradient pass through zero. If we assume the meridional density gradient to pass through zero, the meridional pressure gradient must due to eq. (7) have passed through zero at a lower altitude (Fig. 8), because the meridional temperature and meridional density gradients then are situated at the same side. To explain that the temperature gradient cannot be the first to cross the zero line, we recall that this could happen only if there were a reversal of the vertical motion.

C. 28 **With the existence of only one heating level (earth's surface), a homobaric layer will originate at higher altitudes, namely, above the pleistothermic layer that is present over the Null layer.**

For a thorough investigation of the conditions near the

conditions prevail in the Null layer. The temperature distribution due to vertical motion above the Null layer turns the pressure level on a horizontal line in it, so that it comes to be situated in the horizontal direction. However, this results in a contradiction, the imaginary process showing the horizontal components of ∇T and ∇p to be parallel at any of the altitudes under consideration; that means, there is no vertical motion (eq. (18a)). Irrespective of the contingency of a pressure equalization level $\left(\frac{\partial p}{\partial y} = \frac{\partial p}{\partial x} = 0 \right)$ occurring, it is apparent that the condition cannot be a persistent one: with the horizontal wind speed vanishing and the horizontal temperature gradient not changing significantly, the vertical motion must not only be zero but also change its sign. This is a self-contradiction — a vertical

motion reversal annuls the temperature field which is a necessity of the existence of a pressure equalization level and consequently, of the reversal itself. It becomes clear again that with the existence of only one heating level, the pressure equalization level cannot be persistent.

However, the homobaric layer ($\partial p / \partial y = 0$) can be explained without any contradiction by means of the change in the wind direction with increasing height.

On account of the small value of $\partial p / \partial y$ the components of the disturbances dominate in a certain range below and above the homobaric layer. This causes the wind in this range to shift eastward relatively fast without reaching zero. The vertical motion keeps always its sign, but does not reach zero in the homobaric layer. The direction of the vertical motion is given by the vectorial form of eq. (18a). In the case of the disturbances having opposite x-components, analagous conditions with opposite vertical motions apply. The turn of the wind result from the thermal wind equation (eq. (13) expanded to cover the horizontal wind).

$$\frac{\partial \mathbf{v}_h}{\partial z} = \frac{R}{fp} (\nabla T \times \nabla p)_h$$

C. 29 **The vertical motion does not reverse within a homobaric layer.**

Here it is necessary for the first time to consider the turn of the horizontal wind with height, as has been repeatedly postulated by *H. Dahler* (e.g. [12]). A subsequent, essential statement results regarding the Null layer. Using the vectorial approach, $\mathbf{v}_h = v_h \mathbf{s}$ causes the change with height in the horizontal wind to be

$$\frac{\partial \mathbf{v}_h}{\partial z} = \frac{\partial v_h}{\partial z} \mathbf{s} + v_h \frac{\partial \mathbf{s}}{\partial z} \tag{34}$$

This has been based on natural coordinates, where $\mathbf{s} \times \mathbf{n} = \mathbf{k}$. $\mathbf{s}^2 = 1$, hence

$$\mathbf{s} \cdot \frac{\partial \mathbf{s}}{\partial z} = 0 \tag{35}$$

Thus, $\mathbf{s}$ must be perpendicular to $\partial \mathbf{s} / \partial z$. (34) therefore represents the decomposition of $\partial \mathbf{v}_h / \partial z$ into two components perpendicular to each other, which both must become zero to cause $\partial \mathbf{v}_h / \partial z$ to vanish.

When defining $\partial \alpha / \partial z$ as a turn of the wind with height (positive for shift in a counterclockwise direction), $\partial \mathbf{s} / \partial z$ can be designated by

$$\frac{\partial \mathbf{s}}{\partial z} = \mathbf{n} \frac{\partial \alpha}{\partial z} \tag{36}$$

As for $\partial \alpha / \partial z$, the following relationship holds which is known in theoretic meteorology: —

$$\frac{\partial \alpha}{\partial z} = - \frac{gR}{f^2 p \, v_h^2} (\nabla T \times \nabla p)_{z\text{-comp.}} \tag{37}$$

This is found from the statement $\alpha = \operatorname{arc\,tg} v_y / v_x$ by introducing in it the velocity components from eq. (4). Replacing the above vector product by that from eq. (18a) renders

$$\frac{\partial \alpha}{\partial z} \mathbf{k} = - \frac{g}{v_h^2 f T} \left(\frac{\partial T}{\partial z} + \Gamma \right) v_z \mathbf{k} \tag{38}$$

According to definition, the following holds in the Null layer:

$$v_z = 0; \quad \frac{\partial v_z}{\partial z} \neq 0$$

which by use of eq. (38) reads:

$$\frac{\partial \alpha}{\partial z} = 0; \quad \frac{\partial^2 \alpha}{\partial z^2} \neq 0$$

A rotation vector of the horizontal wind is introduced, $\mathbf{d} = \frac{\partial \alpha}{\partial z} \mathbf{k}$ which according to (38) is proportional to the vertical motion,

$$\mathbf{d} \sim - v_z \mathbf{k}$$

$\mathbf{d}$ stands for the real shift of the horizontal wind due to

$$v_h \frac{\partial \mathbf{s}}{\partial z} = \mathbf{d} \times \mathbf{v}_h$$

Hence, $\mathbf{v}_h$ shifts in a clockwise direction with height if $\mathbf{d}$ shows downward, and conversely.

Table 1 offers a view of the connections between v_z, $v_z \mathbf{k}$, $\partial \alpha / \partial z$, $\mathbf{d}$ as regarded jointly with the Null layer.

Table I

$\frac{\partial v_z}{\partial z} > 0$; $\frac{\partial^2 \alpha}{\partial z^2} < 0$				$\frac{\partial^2 \alpha}{\partial z^2} > 0$; $\frac{\partial v_z}{\partial z} < 0$				Range
v_z	$v_z \mathbf{k}$	$\frac{\partial \alpha}{\partial z}$	$\mathbf{d}$	v_z	$v_z \mathbf{k}$	$\frac{\partial \alpha}{\partial z}$	$\mathbf{d}$	
>0	↑	<0	↓	<0	↓	>0	↑	above Null layer
0	0	0	0	0	0	0	0	within Null layer
<0	↓	>0	↑	>0	↑	<0	↓	below Null layer

Notice that Table 1 is valid only with respect to the Northern Hemisphere. As f shows an opposite sign in the Southern Hemisphere, eq. (38) indicates that $\partial \alpha / \partial z$ gets the sign of v_z and consequently, $v_z \mathbf{k}$ and $\mathbf{d}$ are parallel to each other.

The only presupposition we have employed in deducing Table I is the reversal of the vertical motion. No use has been made of the second Null layer condition, viz., the existence of an extreme of the horizontal wind. The behavior of the horizontal wind thus renders a new characteristic of the Null layer:

C. 30 **In the Null layer, the speed of the horizontal wind shows a maximum, and the wind direction an extreme.**

This means that the two members at the right hand side of eq. (34) become zero individually, and the horizontal wind vector consequently shows an extreme. The case of the minimum of the wind speed which eq. (34) further embodies will be discussed in a later chapter.

*

Let us take a view of the situation above the homobaric layer. After having passed through zero, the meridional pressure gradient rises with increase in height towards the positive values to become again the dominant component of the horizontal pressure gradient. For clearness' sake our consideration will therefore be confined to the (y, z)-plane. $\partial T / \partial y$ keeps its sign also above the homobaric layer. Since $\partial p / \partial y$ changes to the positive side (east wind), and increases, the meridional density gradient must due to eq. (7) cross zero at a specific height.

C. 31 **If there is only one heating level, a homopycnic layer is present above the homobaric layer.**

Evaluation of the three meridional gradients at a position somewhat above the homobaric layer shows the following. $\partial \varrho / \partial y$ is negative and its value decreases with height; $\partial T / \partial y$ is positive and decreases with height, too; $\partial p / \partial y$ is positive and increases with height. Thus, on the whole, the prevailing conditions are analogous to those on the earth's surface in set III, except that the signs of the individual gradients are opposite to those in the latter case. Conclusions in respect of the conditions above the homobaric layer therefore can be adequate to those in respect of set III.

C. 32 **A Null layer with a maximum value of the east wind originates over the above-deduced homopycnic layer where the positive meridional pressure gradient shows a maximum value.**

This Null layer again is connected with a change in the vertical motion and, consequently, in the meridional temperature gradient.

When extending these investigations to still higher altitudes, and assuming the earth's surface to be the sole heating level, alternating zero and extreme conditions of the three gradients will be found which, as before, are always situated at different heights. But, the effects of the heating of the earth's surface weakening with height, the amplitudes of the gradients "swinging" around the zero line must decrease with height.

C. 33 **With the existence of the earth's surface as the sole heating level, regions of west and east wind alternate in the vertical, wherein the wind maximum layers represent Null layers. Separation of the individual regions is by homobaric layers.**

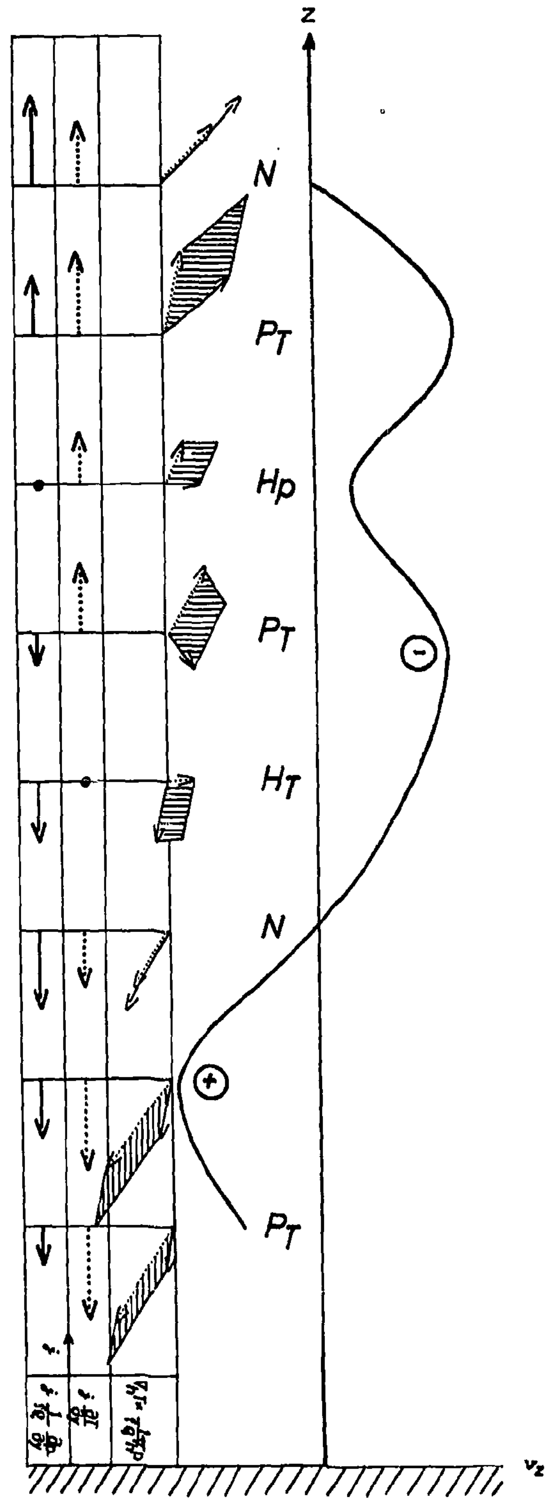


Fig. 10: The large-scale vertical motion in a swinging system

Fig. 9 presents the conditions resulting from the effects of the earth's surface as sole heating level.

The existence of only one heating level causes an atmospheric structure that, as regards the vertical behavior of the meridional gradients, is similar to an oscillation. It is apparent from Fig. 9 that the $\partial T/\partial y$ curve, in particular, cannot be described by a simple sine curve, as due to the behavior of the vertical motion, two pleistothermic layers with identical sign of $\partial T/\partial y$ result between two Null layers.

Fig. 10 shows particular details of the mutual connections between geostrophical wind, the change of it with height and the large-scale vertical motion; the meridional components of $\nabla_h p$ (for v_z) and $\nabla_h T$ have been regarded as dominating and therefore they were additionally plotted. The mathematical basement is given by eq. (18a), (37), (3) resp. the equation of the geostrophic horizontal wind proceeded from eq. (4)

$$\mathbf{v}_h = \frac{1}{f\varrho}\,\mathbf{k} \times \nabla_h p$$

Corresponding to that the value and direction of the geostrophic horizontal wind is given by $1/f\varrho\ \nabla_h p$, whilst the plane between the vectors $\nabla_h T$ and $1/f\varrho\ \nabla_h p$ multiplied by the factor $(\Gamma + \partial T/\partial z)$ represents the vertical motion. To the figure itself it is to say, that the vertical sequence of the arrows is given at specific heights but there the arrows are shown in the horizontal plane with the above definated coordinate system. The altitudes of maximum vertical motion are marked by the sign of v_z which holds for a point within the northern part of the system high/low latitudes being characterized by ascending air in the lowest part of the dynamical system.

Fig. 3 shows the cases from chapter 5 to undergo an obligatory sequence. Fig. 9 indicates that this obligatory sequence is present also in the "swinging" atmosphere along the vertical axis, in the sense that with increasing altitude, the sets from Fig. 3 are passed through from the right to the left. The individual sets are separated from one another by Null layers and by layers of vanishing meridional gradients of pressure, temperature and density (i. e. by layers of vanishing meridional gradients in the p- and z-systems).

C. 34 **In case of the existence of a sufficiently strong heating level the vertical distribution of the meridional gradients of pressure, temperature and density, as well as of the zonal wind and the vertical motion possesses the form of an oscillation with the amplitude decreasing upward. In moving upward, the sets deduced in chapter 5 occur repeatedly in the obligatory sequence of (III), II, Ib, Ia; III, II etc. The undisturbed conditions involve the bottom sets II and III.**

III The Upper Boundary of the Ozone Layer as Heating Level

Subpart A: Summer

The purpose of this Part is to investigate the conditions to arise from assuming a heating level to exist in the free atmosphere, namely, the upper boundary of the ozone layer. Again, empirical data will be neglected. The existence of another atmosphere system for which the earth's surface is a heating level will be disregarded up to Part IV, when the results to forthcome from this will be combined with those obtained in Part II.

Chapter 13: The Meridional Gradients in the Heating Level

The upper region of the ozone layer absorbs almost the whole of the sun's longwave ultraviolet radiation. The quantity of this absorption as a function of both the geographic latitude and the season is placed in proportion to the solar radiation values at the upper limit of the atmosphere as found by Milankovitch [13]. The absorption of the solar ultraviolet rays causes a pleistothermic layer to develop in the upper border of the ozone layer. In summer the polar region is conveyed more solar radiation than the low latitudes (see Fig. 11), which renders $\partial T/\partial y$ positive then.

Heating is not confined to the heating level but can be supposed to extend upward and downward in the course as a rigid level. Owing to the heated spaces near the ozone layer upper border the constant pressure levels will rise in the high latitudes. The Coriolis force prevents the air masses from flowing toward the lower pressure.

C. 35 **In the upper border of the ozone layer there is in summer a meridional pressure gradient which is directed toward the polar region and according to eq. (3) is connected with east wind.**

In order to theoretically find the sign of $\partial \varrho/\partial y$ in the heating level, we must, as a starting point, use the assumption that below the heating level, the meridional

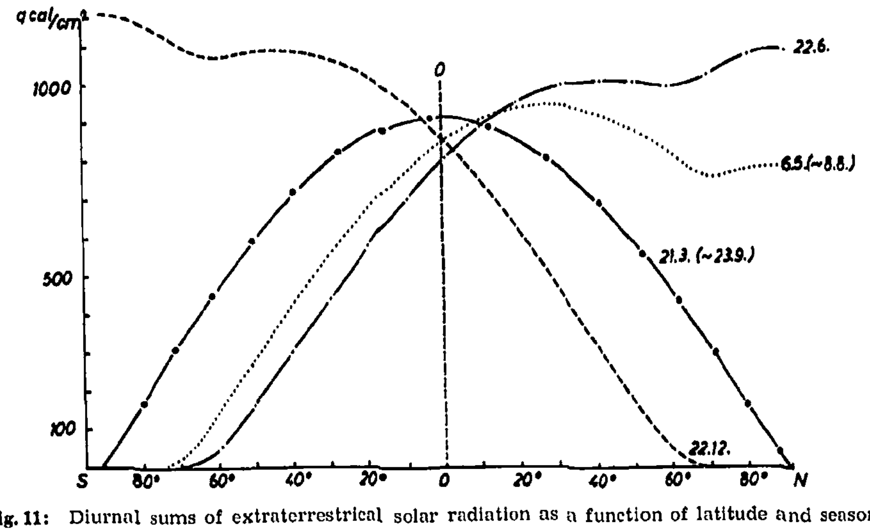


Fig. 11: Diurnal sums of extraterrestrical solar radiation as a function of latitude and season

of time, with $\partial T/\partial y$ decreasing as its distance from the heating level increases.

In order to say which sign $\partial p/\partial y$ has in the heating level we avail ourselves of one fact from the lower atmospheric regions: the existence of the earth's surface pressure gradient caused by it fades to become zero at a specific altitude (homobaric layer).

If the effects of the heating level had a merely static nature, the homobaric level would at the same time have to be a homothermic one. But so that dynamic

effects may not be precluded from the beginning, the two layers must not be assumed to coincide.

We for the first time find the temperature to rise with increase in height. To simplify matters $\partial T / \partial z = \gamma$ is supposed to be independent of height over a given place, and to enable a finite temperature equalization to occur, $\partial T / \partial z = \gamma(y)$ has to be a function of the latitude.
From eq. (1) and (2) it follows that

$$(39) \qquad dp = -\frac{gp}{RT}\,dz$$

With γ constant (here positive), the temperature at altitude z ($\leqq H$) over a given place is

$$(40) \qquad T = T_H - \gamma (H - z)$$

for $z = 0$, that means in the pressure equalization level (p_o = constant), holds $T = T_o$.

We introduce eq. (40) into (39) and take the integral between the limits 0 and H:

$$\ln \frac{p_H}{p_o} = -\frac{g}{R\gamma} \Big[\ln (T_o + \gamma z) \Big]_o^H$$

For the sake of simplicity the indices "H" will in the following be left out. Substituting the limits and making allowance for eq. (40) finally renders:

$$(41) \qquad p = p_o \left(\frac{T}{T - \gamma H} \right)^{-\frac{g}{R\gamma}} = p_o \left(1 - \frac{\gamma H}{T} \right)^{\frac{g}{R\gamma}}$$

This equation expresses the pressure as a function of the temperature at the height of the heating level. Finding the partial derivative with respect to y, the sign of $\partial \varrho / \partial y$ can be determined on the basis of eq. (7).

To differentiate eq. (41), it is stated:

$$\left(1 - \frac{\gamma H}{T} \right) = a(y)\,; \qquad \frac{g}{R\gamma} = b(y)$$

then, the following relationship is employed:

$$(42) \qquad \left(a(y)^{b(y)} \right)' = a(y)^{b(y)} \left(b' \ln a + a' \frac{b}{a} \right)$$

By placing a (y) and b (y) in (42) the following is obtained:

$$(43) \qquad \frac{1}{p} \frac{\partial p}{\partial y} = -\frac{g}{R\gamma^2} \frac{\partial \gamma}{\partial y} \ln \left(1 - \frac{\gamma H}{T} \right) + \frac{g}{R\gamma} \frac{\left(\frac{\gamma H}{T^2} \frac{\partial T}{\partial y} - \frac{H}{T} \frac{\partial \gamma}{\partial y} \right)}{\left(1 - \frac{\gamma H}{T} \right)}$$

$\gamma H / T$ may be assumed to be considerably smaller than 1, thus

$$1.) \quad \ln \left(1 - \frac{\gamma H}{T} \right) \approx -\frac{\gamma H}{T} \qquad 2.) \quad \frac{1}{1 - \frac{\gamma H}{T}} \approx 1 + \frac{\gamma H}{T}$$

also, the terms possessing $(\gamma H / T)^2$ can be neglected, which renders

$$(44) \qquad \frac{1}{p} \frac{\partial p}{\partial y} = \frac{gH}{RT} \frac{1}{T} \frac{\partial T}{\partial y} - \frac{\partial \gamma}{\partial y} \left(\frac{gH^2}{RT^2} \right)$$

This equation is introduced into eq. (7), while RT/g is replaced by H_h (which imagine as the height of a homogeneous atmosphere having the basic temperature T); hence,

$$(45) \qquad \frac{1}{\varrho} \frac{\partial \varrho}{\partial y} = \frac{1}{T} \frac{\partial T}{\partial y} \left(\frac{H}{H_h} - 1 \right) - \frac{H}{H_h} \frac{H}{T} \frac{\partial \gamma}{\partial y}.$$

$\partial \gamma / \partial y$ can by means of eq. (40) be expressed as

$$\frac{\partial \gamma}{\partial y} = \frac{1}{H} \left(\frac{\partial T}{\partial y} - \frac{\partial T_o}{\partial y} \right)$$

which finally renders the following, illustrative form of eq. (45):

$$(46) \qquad \frac{1}{\varrho} \frac{\partial \varrho}{\partial y} = \frac{1}{T} \left(\frac{H}{H_h} \frac{\partial T_o}{\partial y} - \frac{\partial T}{\partial y} \right)$$

From this, the sign of $\partial \varrho / \partial y$ in the heating level can be found

$$(47) \qquad \frac{\partial \varrho}{\partial y} \gtreqless 0, \quad \text{if} \quad \frac{\partial T}{\partial y} \lesseqgtr \frac{H}{H_h} \frac{\partial T_o}{\partial y}$$

In deriving this formula our assumption has been that a pressure equalization level exists at a finite altitude below the heating level; another has been that the vertical temperature gradient remains constant over a given place, but not in the y-direction. These restrictions — which certainly are not weighty — allow conclusions to be drawn in the heating level with respect to the sign of $\partial \varrho / \partial y$. Generally speaking a negative sign of $\partial \varrho / \partial y$ results if $\partial T_o / \partial y = 0$ (static conditions). Chapter 14 furnishes a positive sign of $\partial T_o / \partial y$ and also an additional possibility of variation in H. Provided the temperature conditions of the heating level and of the homobaric level ($\partial T_o / \partial y > 0$) remain constant, a remarkable result is obtained: The smaller (larger) H, the more likely a negative (positive) $\partial \varrho / \partial y$. If on the other hand both H and $\partial T / \partial y$ keep constant, the probability of a positive $\partial \varrho / \partial y$ increases with increase in $\partial T_o / \partial y$.

C. 36 **In the summer ozone heating level, when only considering its static effects, the air density must increase from high to low latitudes. When taking into account also the dynamic effects, the sign of $\partial \varrho / \partial y$ in the heating level depends upon the width of the vertical range on which influence is exerted by the temperature differences in the heating level.**

Eq. (47) thus does not provide an unambiguous statement regarding the sign of the meridional density gradient at the upper boundary of the ozone layer and consideration will have to be given to both possibilities.

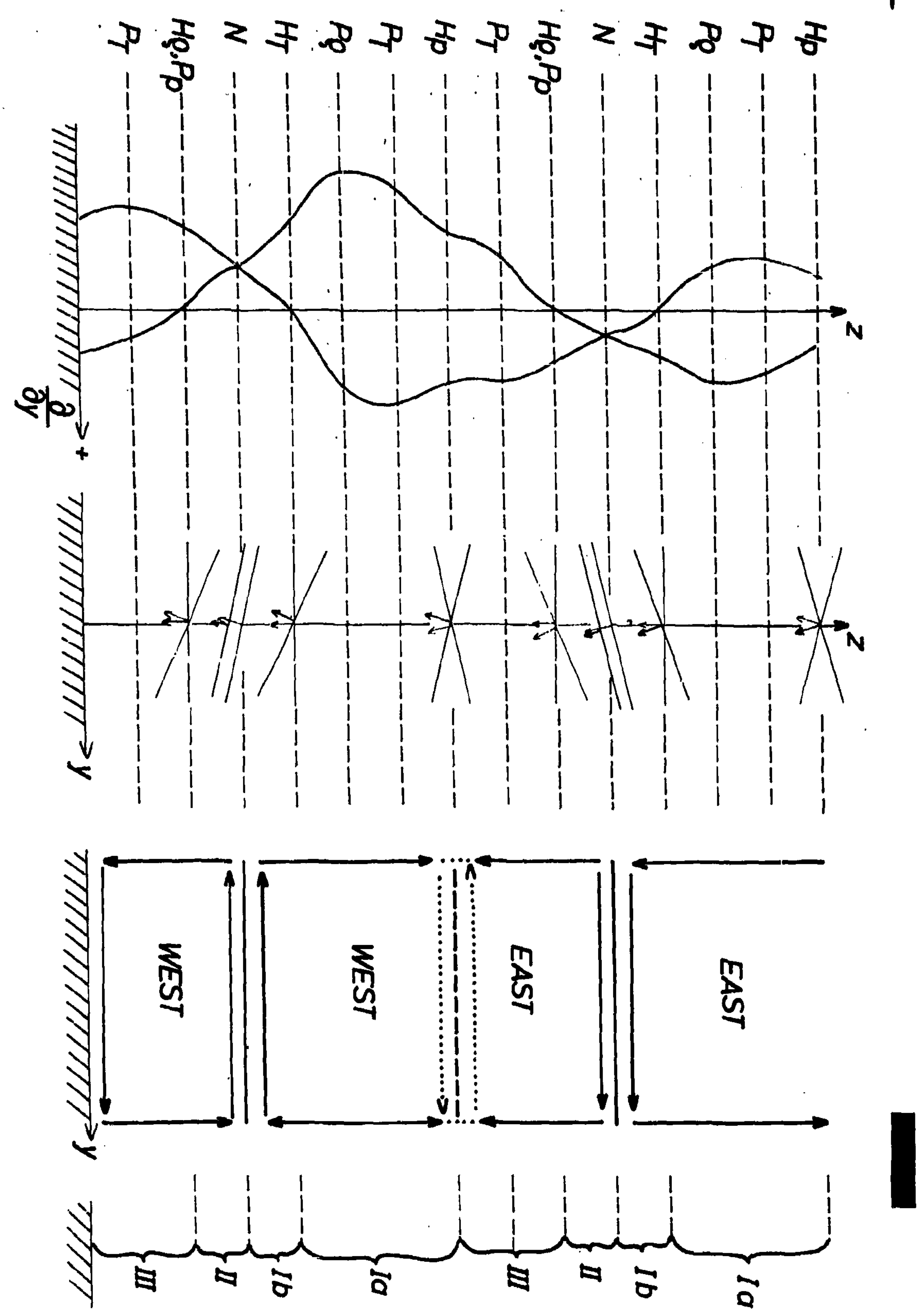


Fig. 9: The swinging system above a heating level

Chapter 14: The Conditions above the Heating Level

Above the heating level $\partial T/\partial z$ is again negative, and we may expect the same layers to occur as above the other heating level, the earth's surface. In the ozone heating level, a positive $\partial\varrho/\partial y$ corresponds to the bottom set II, a negative $\partial\varrho/\partial y$ to the bottom set III (chapter 5), at which the signs of the meridional gradients now are opposite to those on the earth's surface.

C. 37 **In summer there prevails above the ozone heating level the same sequence of layers as above the earth's surface. The appertaining bottom sets are not required to be the same.**

In particular, there is above the heating level a persistent east wind maximum which represents a Null layer with vertical motion reversal and nongradientic mass flow from low to high latitudes. In the latter Null layer the meridional density gradient is positive — independent of its original value in the heating level — for the reason that the meridional gradients of the three meteorologic parameters have to have the like sign. Thus it is in the Null layer, for the first time, that a definite statement as to the sign of $\partial\varrho/\partial y$ is possible. Owing to the effects of the Null layer new angles arise regarding the sign of $\partial\varrho/\partial y$ in the heating level. The nongradientic mass flow makes the meridional pressure gradient intensify at the upper boundary of the ozone layer, which, with $\partial T/\partial y$ remaining unchanged, leads to a shift of $\partial\varrho/\partial y$ in the direction of positive values. The influence of the vertical motion that is caused by the Null layer renders the positive value of $\partial T_0/\partial y$, which has already been used in the previous chapter. This effect acts in the same direction in respect of $\partial\varrho/\partial y$ in the heating level. The above considerations may prove to be of significance when regarding the time-dependent temperature distribution within the heating level.

Chapter 15: The Conditions below the Heating Level

Consideration will now be given to the conditions below the heating level, and, first, in it $\partial\varrho/\partial y$ shall be positive. The temperature and pressure differences proceed, weakening, to the lower altitudes; $\partial\varrho/\partial y > 0$ requires according to eq. (1) $\partial p/\partial y$ to rise downward. But this cannot take place continuously since it is impossible for the potential energy of the horizontal pressure distribution, being a function of the strength of the heating level, to rise infinitely. A maximum of the horizontal pressure gradient occurs, at which $\partial\varrho/\partial y$ is equal to zero, and changes its sign. $\partial p/\partial y$ now decreases with decrease in altitude.

C. 38 **With $\partial\varrho/\partial y$ positive in the upper border of the ozone layer, a homopycnic layer develops below same in summer.**

$\partial T/\partial y$, too, decreases with decreasing altitude; therefore, if $\partial\varrho/\partial y$ is negative with its value increasing, $\partial p/\partial y$ finally must change over to the direction of $\partial\varrho/\partial y$. That is to say:

C. 39 **A homobaric layer occurs below the homopycnic layer.**

Below it, the west wind increases with decrease in altitude and a Null layer with west wind finally develops, in which ∇T is antiparallel to both ∇p and $\nabla\varrho$ because of $\partial T/\partial z > 0$. Hence, a pass through zero by the meridional temperature gradient occurs somewhat below the Null layer.

C. 40 **If $\partial T/\partial z$ is positive, the homothermic layer which belongs to a Null layer is situated somewhat below this.**

In the case of the existence of a Null layer, the same conditions hold good for a negative $\partial\varrho/\partial y$ as they do for a positive one below the homopycnic layer. If the energy transformed in the heating level is not so strong as to create a Null layer, the three gradients will jointly fade until reaching zero. This is a borderline case not to be discussed here.

So far the space below the heating level has been impliedly understood to be sufficient to cause the swinging behavior of the three meridional gradients of p, ϱ and T. What has been said in Part II with regard to the development of layers of extreme meridional gradients as a consequence of the large-scale vertical motion applies analogously in this part. The positive sign of $\partial T/\partial z$ is significant only for the position of the homothermic layer which is connected with the Null layer, and does not affect the relative positions of the pleistothermic and pleistopycnic layers (see eq. (32)), because in the free atmosphere, $\partial T/\partial z$ is always smaller than g/R. Similarly, the relative positions of the other 'pleisto' and 'homo' layers are independent of the sign of the vertical temperature gradient.

The sets derived in chapter 5 have been applicable to a negative vertical temperature gradient which has been the only one possible in case of the existence of the earth's surface as a heating level. However, a positive $\partial T/\partial z$ requires a modification to be made to the limits (Fig. 3), B changing its sign. The new limits can be found by analogy with the calculations of chapter 5. Fig. 12 shows the new conditions as they correspond to Fig. 3, and also applies to a negative $\partial p/\partial y$ (west wind). Formally, set Ib of Fig. 3 (wind and meridional pressure gradient decreasing with height, same sign of the three meridional gradients) is superseded by a set to be termed II* (wind increasing while pressure gradient decreasing with height, sign of meridional temperature gradient opposite to those of meridional pressure and meridional density gradients). This designation has been chosen so as to enable those of the earlier sets to be retained.

Fig. 13 illustrates, by analogy with Fig. 9 (heating level on earth's surface), the conditions prevailing if there is only one heating level in the free atmosphere.

The importance of the large-scale vertical motion for the development of pleistothermic and pleistopycnic layers has been fully discussed in Part II; it has also been shown there that in the space where all the three horizontal gradients possess the like sign, there cannot be a pleistopycnic layer, whereas, theoretically, there can be a minimum layer. The latter has not been taken into account on Fig. 13.

Comparing the results attained in chapter 14 with those of chapter 15 permits the following conclusion:

C. 41 **The homopycnic layer which is between a Null layer and the homobaric layer below same, is situated underneath the heating level, provided that the meridional gradients of p, T and ϱ have the like sign. It is situated above the heating level, if the meridional density gradient is opposite to the two others.**

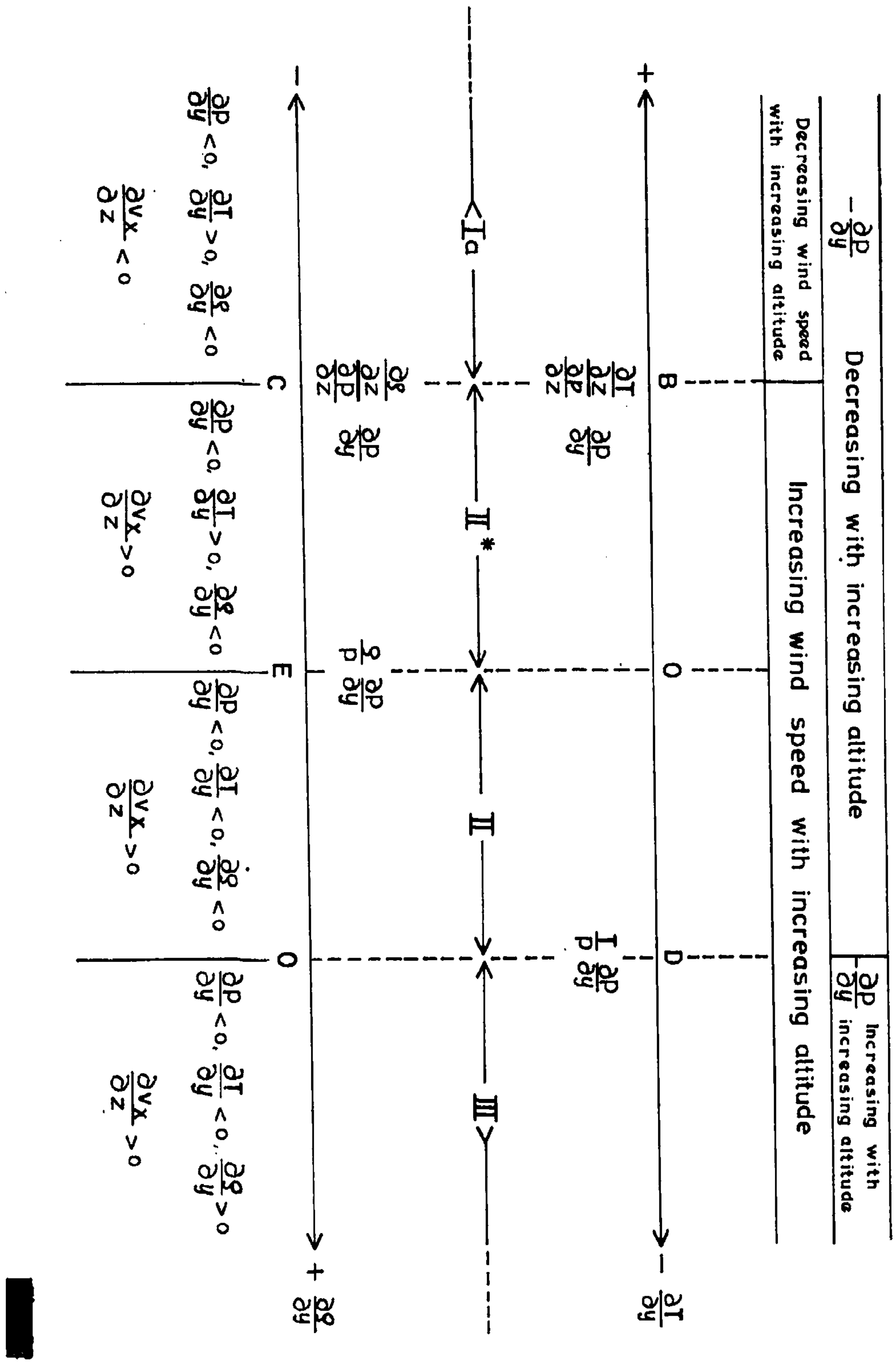


Fig. 12: The system of the sets for $\partial T/\partial z > 0$

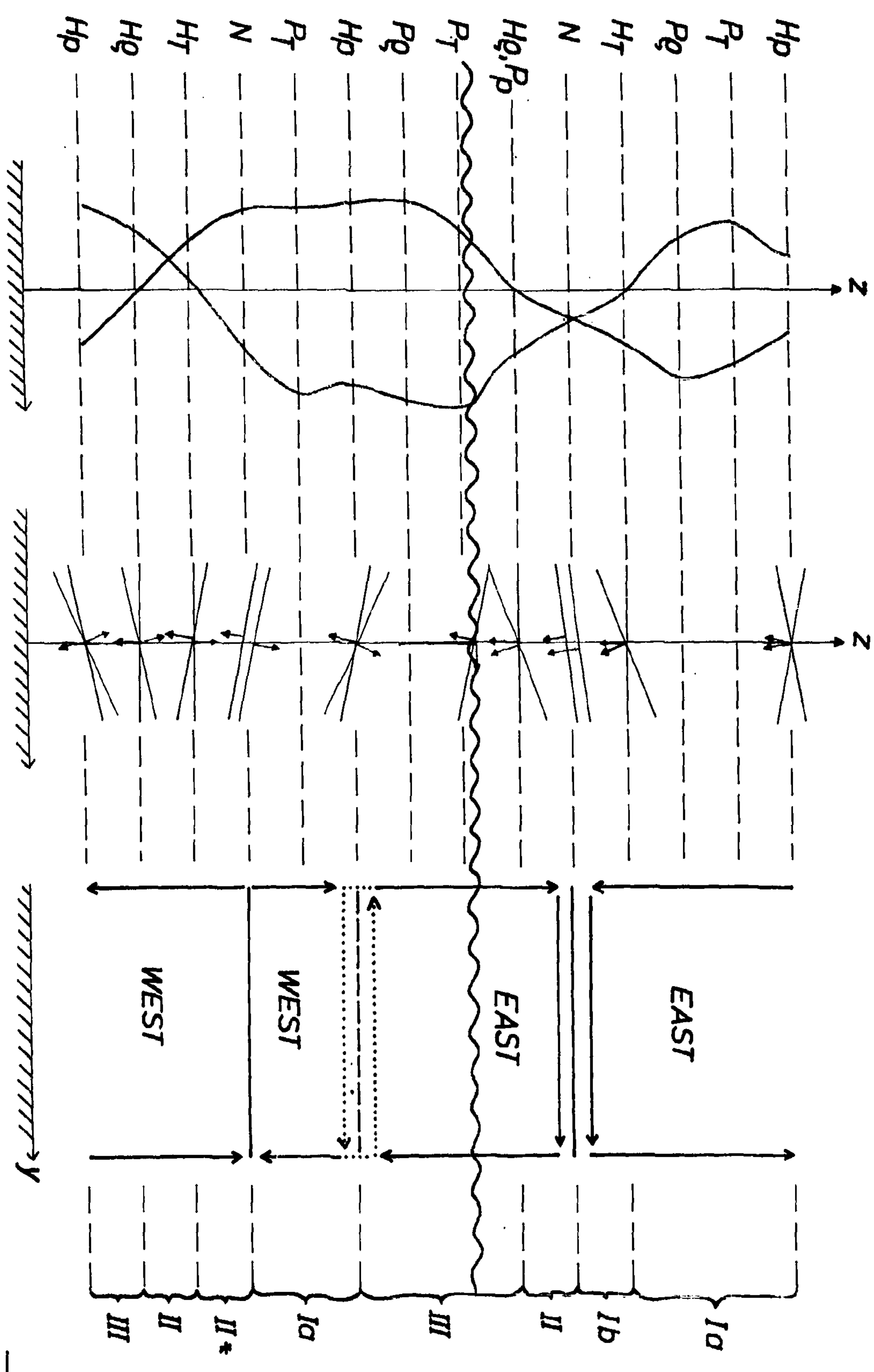


Fig. 13: The swinging system on both sides of a heating level

Subpart B: Winter

Chapter 16: The Effects of the Upper Boundary of the Ozone Layer in Winter

It is apparent from Fig. 11 that in winter the low latitudes receive more solar radiation than the high latitudes; the difference in the radiant energies conveyed is considerably greater than in summer. From this we may infer that a greater meridional temperature difference exists at the upper boundary of the ozone layer. What has been said in Subpart A applies equally to the case of winter when reversing the signs of the meridional gradients. In particular, the vertical sequence of the deduced layers holds good here too. There is, however, one remarkable difference between summer and winter in the nontropical latitudes on account of the greater difference in the meridional temperatures within the heating level: The amplitudes of the "swinging" gradients are larger, that means, the pleisto-layers are more marked. On the whole, the meridional circulation may be assumed to be more intense in winter. Fig. 11 also indicates that the summer circulation covers a shorter period of time than that in winter.

A representation of winter conditions would be like a mirrored Fig. 13 and has therefore been omitted.

IV The Joint Action of the Heating Levels

Chapter 17: Comparison of the Two Individual Circulation Systems

The most noticeable difference between the circulation systems that are produced by those two heating levels is that in the lower system, the circulation does not change its direction during the course of the year, whereas in the upper system, it is opposite during the extreme seasons. This difference renders a natural principle for an arrangement in this part.

As would become clear from Figs. 9 and 13, the vertical sequence of the layers of marked characteristics is the same in both systems of circulation, except that in the range where the vertical temperature gradient is positive, the Null layer is above the appertaining homothermic layer. In the vicinity of the Null layer, the sequence of those cases therefore is dependent on the sign of the vertical temperature gradient: Set Ib occurs only where $\partial T / \partial z$ is negative, set II* where it is positive.

C. 42 **For a "swinging" atmosphere the vertical sequence of the sets above the upper boundary of the ozone layer is the same as that above the earth's surface ($\partial T / \partial z < 0$). Below the upper boundary of the ozone layer ($\partial T / \partial z > 0$), set Ib is, formally, superseded by set II*, which is below the Null layer according to C. 40.**

The slopes of the surfaces on Fig. 13 offer further possibilities of the thermal structure of the pressure systems in the systems of z and p. Chapter 6 has indicated that one and the same high-pressure area (low-pressure area) may be cold (warm) in the p-system while warm (cold) in the z-system, with the wind speed decreasing with height (set Ib).

The statements that have been made in the systems of p and z relative to the thermal structure of pressure systems will, however, change if the vertical temperature gradient is positive; then, the wind speed will increase with height (set II*).

C. 43 **It is only with a positive vertical temperature gradient that in the z-system the high-pressure area can be cold and the low-pressure area warm, though the wind speed increases with height. It is only with a negative vertical temperature gradient that in the z-system the high-pressure area can be warm and the low-pressure area cold, though the wind speed decreases with height.**

Chapter 18: Possible Forms of Vertical Structure of the Atmosphere as Resulting from the Joint Action of Two Heating Levels

In Parts II and III the effects of only one heating system in the atmosphere have been deduced, our investigations now are to ascertain those of the simultaneous action of both heating levels. Theoretically, a sole heating level exerts an influence upon the whole of the atmosphere; its influence, however, decreases as the distance from it increases. In order to enable any results to be attained, it is assumed that the effects of the one heating level are negligible in the vicinity of the other.

To think that each of the two independently "swinging" heating level systems could be cut off at a specific place, and the two put abruptly together at such "cut-off" places, would be physically wrong. In general, this would mean leaps of the meteorologic parameters. For simplification, and imagining a vertical range to exist between the two heating levels, within which the supremacy shifts from the one system to the other, the following definition is made: Above a specific height H_a (lower boundary of the undisturbed, upper system), only the upper system is significant; below a specific height H_b (upper boundary of the undisturbed, lower system; $(H_b < H_a)$, only the lower system. Transition from the one system to the other takes place between H_b and H_a; in this range, there is not a marked dynamic layer, which might have developed due to the effects of one system only.

For the range of transition, specific border conditions of the parameters $\partial p / \partial y$, $\partial T / \partial y$, $\partial \varrho / \partial y$, v_x and v_z are given at H_a and H_b. We now ask for the possible ways of transition of those into one another. The variety of the transition that would be possible formally, is, however, limited by the condition that the final state within the range of transition must be persistent; in other words, it must not be connected with any processes trying to destroy it.

Occasionally the vertical distribution of the meteorologic parameters has in the previous chapters been compared to an oscillation, the legitimation of this having been derived by replacing the time-coordinate of common oscillations by the z-coordinate. Comparison to a "frozen" wave may therefore be still more to the point.

The oscillation-like behaviors of the meridional gradients as well as of v_x and v_z are, finally, due to the behavior of v_z, which in turn is connected with the field of the zonal wind. It therefore seems to be legitimate from a physical point of view to regard both the wind components as representative of the phenomenon as a whole.

In order to find out the possible ways of behavior of the wind components within the transition range, consideration is given to the persistent combinations of v_x, $\partial v_x / \partial z$ and v_z as obtained in chapters 12 and 15. It has proved there that, while $\partial v_x / \partial z$ keeps the same sign, the signs of v_z in the high and low latitudes respectively are opposite. Therefore, without impairing the universality of the forthcoming results, we need not regard other than the conditions at the low latitudes.

The ocurring combinations are as follows:

Increase of west wind with height and descending air $+\,\mathrm{W}\downarrow$

Decrease of west wind with height and ascending air $-\,\mathrm{W}\uparrow$

Increase of east wind with height and ascending air $+\,\mathrm{E}\uparrow$

Decrease of east wind with height and descending air $-\,\mathrm{E}\downarrow$

The other combinations that are conceivable formally, are not persistent in the low latitudes. Any of the above four combinations can occur at H_a as well as at H_b, and the 16 linkages that are possible between them are given in the table below.

	a	b	c	d
1	$\frac{-\mathrm{W}\uparrow}{+\mathrm{W}\downarrow}$	$\frac{-\mathrm{W}\uparrow}{-\mathrm{W}\uparrow}$	$\frac{-\mathrm{W}\uparrow}{-\mathrm{E}\downarrow}$	$\frac{-\mathrm{W}\uparrow}{+\mathrm{E}\uparrow}$
2	$\frac{+\mathrm{W}\downarrow}{+\mathrm{W}\downarrow}$	$\frac{+\mathrm{W}\downarrow}{-\mathrm{W}\uparrow}$	$\frac{+\mathrm{W}\downarrow}{-\mathrm{E}\downarrow}$	$\frac{+\mathrm{W}\downarrow}{+\mathrm{E}\uparrow}$
3	$\frac{+\mathrm{E}\uparrow}{+\mathrm{W}\downarrow}$	$\frac{+\mathrm{E}\uparrow}{-\mathrm{W}\uparrow}$	$\frac{+\mathrm{E}\uparrow}{-\mathrm{E}\downarrow}$	$\frac{+\mathrm{E}\uparrow}{+\mathrm{E}\uparrow}$
4	$\frac{-\mathrm{E}\downarrow}{+\mathrm{W}\downarrow}$	$\frac{-\mathrm{E}\downarrow}{-\mathrm{W}\uparrow}$	$\frac{-\mathrm{E}\downarrow}{-\mathrm{E}\downarrow}$	$\frac{-\mathrm{E}\downarrow}{+\mathrm{E}\uparrow}$

Table II: Formally possible linkages of the combination types. Combination types assumed to occur at H_a (H_b) are shown above (below) the horizontal line.

The lines of Table II represent the unchanged combination types at H_a, the columns those at H_b. When selecting the linkages that are physically possible, the definition of the transition range as one of the occurrence in it of one marked dynamic layer, if any, must be taken into account. The linkages 1d and 4a would require two N's and, between these, one H_p, to be incorporated in the transition range. The linkages 1c, 2d, 3a and 4b would make necessary one N and one H_p in the transition

range. These linkages, which are merely formal, are inconsistent with the above supposition and must therefore be disregarded. 2c and 3b yield one H_p in the transition range, while 1a and 4d, one N. 2b and 3c yield a dynamic layer which has not so far occurred, and whose essential characteristics originate from the combination types assigned to it. This layer is one of minimum zonal winds with a reversal of the vertical motion; it meets all the requirement of a Null layer, and will below be called "Null layer of 2nd kind" (N_2). The earlier Null layer, which is connected with a maximum of the zonal wind, will below be designated „Null layer of 1st kind" (N_1). The N_2 will be fully discussed in the next chapter.

The linkages 1b, 2a, 3d and 4c are still left for consideration. At these, a marked dynamic layer does not come into existence in the transition range, because the relevant combination types at H_a and H_b are equal.

Table IIa offers a survey of the marked dynamic layers that occur at the respective linkages. The absence of such a layer is denoted by a cipher, an excluded linkage by a cross.

	a	b	c	d
1	N_1	0	×	×
2	0	N_2	H_p	×
3	×	H_p	N_2	0
4	×	×	0	N_1

Table IIa: Marked dynamic layers in the transition range.

Thus it is apparent that within the range of transition, there develops either a Null layer of the 1st or 2nd kind, a homobaric layer, or no marked dynamic layer at all.

Other considerations also lead to the conclusion that the variety of the possible ways of transition can be reduced to a small number of basic forms. We shall below confine ourselves to considering v_x and v_z. For a combined system of two heating levels it is of importance whether v_x respectively v_z passes through zero within the transition range or not. There are, formally, four possibilities, which are included in Table III.

	Pass through zero	No pass through zero	Marked dynamic layer in the transition range
a	—	v_x, v_z	0
b	v_x	v_z	H_p
c	v_z	v_x	N_1 or N_2
d	v_x, v_z	—	×

Table III: Behavior of v_x and v_z in the transition range.

Possibility *a* materializes on the equality at H_a and H_b of the signs of the meteorologic parameters. In this case, which occurs between an N_1 and an H_p, the range of transition does not comprehend a marked dynamic layer. Possibility *b* materializes when the zonal winds at H_a and H_b are opposite to each other; the two wind systems are in the transition range separated by a homobaric layer. There occurs a Null layer, either of the 1st or 2nd kind, in that range if the vertical motions at H_a and H_b are opposite to each other (possibility c), while the zonal winds have the like sign. Possibility *d* cannot materialize, because v_x and v_z cannot change their signs simultaneously; see the investigation in chapter 12 concerning the homobaric layer. Thus the same four forms of transition within the transition range are obtained as above.

These four forms of transition can be included in two virtually different groups. Comparison of them with vertical sections of only one "swinging" system reveals that three of them (N_1, H_p, 0) are contained in the latter, whereas the fourth one (N_2) is not. The abovementioned two groups be given their own names with a view to facilitating the relevant statements below.

1. *When the transition range includes either no marked dynamic layer or an N_1 respectively H_p, the two systems involved are defined as c o n c u r r e n t with one another. The vertical sequence of the marked dynamic layers in the composite system corresponds to that in a sole system.*
2. *When the transition range includes an N_2, the two systems involved are defined as c o u n t e r c u r r e n t to one another. The vertical sequence of the marked dynamic layers in the composite system does not correspond to that of a sole system.*

C. 44 **Between two heating level systems that are capable of swinging, there are virtually different forms of transition which result from concurrent and countercurrent systems respectively.**

As yet the sets defined in chapters 5, 12, 14 and 15 have not been utilized. Notice that the combination types here used can include up to 3 such sets. E. g., + W ↑ can embrace sets II, II* and III, which shows that the essential features of the transition range have already been determined by the combination types, whereas the finer structure offered by those sets appears to be unimportant dynamically. The more complicated way of utilizing the sets rather than the combination types would have yielded identical results. The primary importance attached herein to the wind field is evident also where two systems act jointly.

We have hitherto assumed the range of transition to more or less extend vertically. Now the question arises whether a proper definition can be given to a nearly

two-dimensioned separating layer. There must in the transition range be a level of balance between the effects of the two systems, and there would be nothing more obvious than regard this level as the separating layer. However, our foregoing results do not permit to characterize this layer by means of meteorological parameters. Further, the separating layer could be represented by that level in which the decrease in temperature of the lower system passes over to the increase of temperature of the upper one. Selection of this layer is supported by the following. At first approach, the vertical temperature gradient is directly caused by the heating level. This is illustrated by the temperature arrows (see Fig. 13), which point to the respective heating .evel. Also, the sequence of sets is dependent upon the sign of $\partial T / \partial z$. When using another meteorological quantity to define the separating layer, a set clearly pertinent to o n e of the two systems only (Ib, II*), may happen to become assigned to the other. The heavy disadvantage of using $\partial T / \partial z = 0$ in the definition of the separating layer is that the temperature gradient, which primarily is caused by the heating levels, is modified by the vertical motion. This can subject the $\partial T / \partial z = 0$ level to heavy vertical shifts. Under the circumstances it is not advisable to have recourse to the local, vertical temperature gradient.

The above disadvantage can, however, be largely eliminated by deriving the mean of temperature over a whole coherent system (high latitudes — low latitudes), and forming the vertical gradient of this mean: —

$$T_m = \frac{1}{F} \iint_F T dF, \quad \frac{\partial T_m}{\partial z} = \frac{\partial}{\partial z} \left(\frac{1}{F} \iint_F T dF \right)$$

where F stands for the area that at the respective height is covered by the coherent system. Since the effects of the vertical motion do not hold with $\partial T_m / \partial z$, the definition of the range of transition renders the height where $\partial T_m / \partial z$ becomes zero to be situated within same. The equation of definition, $\partial T_m / \partial z = 0$, yields a layer which is situated at the same height in all latitudes. The separating layer between the two systems generally will not be a horizontal level; therefore, in defining the separating layer over a given place, $\partial T_m / \partial z = 0$ cannot be used but as a first lead.

This makes neccessary the finding of another basis on which to define the separating layer, because later practical use of the results will require matters to be assigned to o n e system. The primary rôle of the wind field has been repeatedly referred to in this chapter; its marked dynamic layers will be used to define the separating layers.

This is not practicable in respect of the linkages 1b, 2a, 3d and 4c of Tables II and IIa (see also possibility *a* of Table III), no marked dynamic layer developing at these in the transition range. For all these linkages, the respective separating layer to be defined is not within the transition range.

It is now possible to give the following definition, which is generally applicable:

> *The separating layer between two heating systems be that marked dynamic layer (H_p, N_2, N_1) which is next to the $\partial T_m / \partial z = 0$ level.*

In case of counterourrence, a separating layer can thus only be represented by an N_2, in case of concurrence, by either an H_p or an N_1.

Chapter 19: Manifestations of the Countercurrent Transition; the Null Layer of 2nd Kind (N_2)

The interpretation of eq. (13) and (18) which are connected with an extreme condition of the wind has as yet been restricted almost exclusively to the wind maximum. A thorough analysis will now be made of the case of the wind minimum also comprised by those equations, and which mathematically is equal to that of the wind maximum. It has appeared from regarding the conditions prevailing on the existence of only one heating level that there is no N_2 in the "swinging" atmosphere; on the other hand, the previous chapter has shown that such a layer develops on the existence of two heating systems that are linked countercurrently.

C. 45 **It is only by the joint action of two heating systems that a Null layer of 2nd kind can develop.**

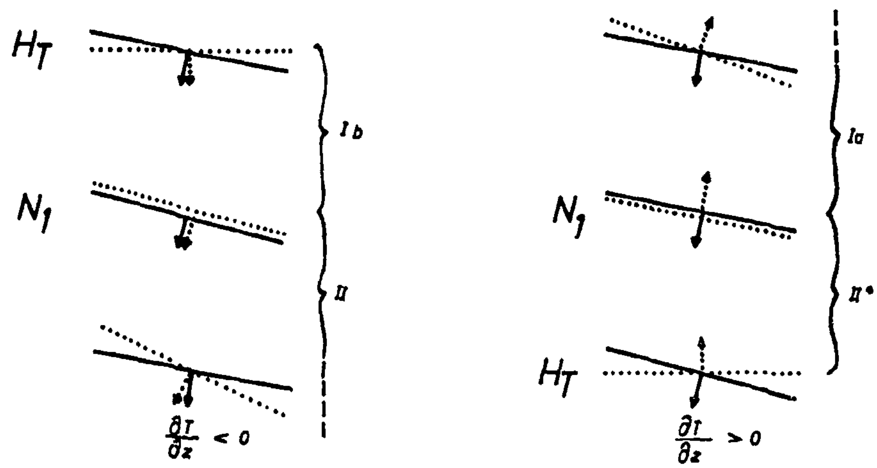


Fig. 14: Couples of sets in connection with an N_1

The investigations relative to the characteristics of a persistent wind minimum layer can be based upon those made in chapter 4. The maintenance of a wind minimum is equivalent to the persistency of the corresponding quartered field of temperature distribution, which in turn can only be due to the relevant field of the large-scale vertical motion.

The quartered temperature field belonging to an N_2 is opposite to that of an N_1. Within the range influenced by an N_2, the vertical motion therefore is convergent in the area of high pressure, while it is divergent in the area of low pressure. For continuity, a nongradientic flow of air masses will take place from the area of high pressure to that of low pressure. Referring to eq. (34) and to C. 30 (chapter 12), we can ascribe the following characteristic to the N_2.

C. 46 **A Null layer of 2nd kind is represented jointly by a persistent minimum of the horizontal wind speed, an extreme of the direction of the wind, a reversal of the large-scale vertical motion together with a nongradientic flow of air masses from the area of high pressure to the area of low pressure.**

Eq. (13) and (18) in conjunction with the latter conclusion render the p, T and ϱ surfaces parallel to one another at each point of the N_2. ∇p and $\nabla \varrho$ are parallel, as they are in the case of the N_1; the direction of ∇T, however, depends upon the sign of $\partial T/\partial z$. With $\partial T/\partial z$ negative, ∇T is parallel to the other two gradients; with $\partial T/\partial z$ positive, it is antiparallel to these. This allows statements to be made regarding the relative position of the N_2 and its appertaining H_T. We are referring to Fig. 14 which holds for the N_1. Due to the fact that the change of wind with height occurring with an N_2 is opposite, the relative slope of the T-surface to the p-surface is opposite as well. From this it can be concluded:

C. 47 **When $\partial T/\partial z$ is negative, the homothermic layer is below the Null layer of 2nd kind to which it belongs; when $\partial T/\partial z$ is positive, it is above. Thus, as compared with the N_1, conditions are reversed.**

The couples of sets on Fig. 14 have proved to be the only ones that can include an N_1. The arrangement of the couples holding for the N_2 on the basis of the foregoing, and in accordance with the definitions of the sets, is inverse to that holding for the N_1:

	$\partial T / \partial z < 0$	$\partial T / \partial z > 0$
N_1	Ib II	Ia II*
N_2	II Ib	II* Ia

In view of the indication in chapter 7 that an N_1 can occur both with and without a homopycnic layer, the relations between the N_2 and the H_ϱ will now be examined. First of all, this requires the possible connections between the extremes of the zonal winds and those of the meridional pressure gradient to be elucidated. As is evident from chapter 5, the occurrence of a wind maximum can be with a maximum of the pressure gradient as well as without, and when there is a maximum of the pressure gradient, it is positioned below the appertaining wind maximum. Though, it is possible theoretically that a maximum of the pressure gradient does not directly entail one of the wind. Due to the basic facts this latter occurrence, too, would cause a wind maximum at not too great a distance from the heating level, as has been shown in chapter 4.

C. 48 **When a minimum of the meridional pressure gradient originates one of the zonal wind, the latter is situated below the former.**

Further, since a minimum of the meridional pressure gradient cannot occur within one heating system,

C. 49 **A persistent minimum of the meridional pressure gradient can be caused only by the joint action of two systems.**

Possibility 2c of Fig. 15 yields a wind minimum whenever the nearest marked dynamic layer of the lower system is an N_1. In this case, 2c can be regarded as the upper part of 2b.

Only those possibilities which render a wind minimum are of interest here. The range of consideration is that between two N_1s having the same direction of wind (i. e., those with an N_2 between them). Due to conclusions C. 2, C. 3, C. 48 a homopycnic layer (if it exists) is situated below the pertinent N_1 respectively above the pertinent N_2. Thus between an N_2 and the N_1 below it there cannot be a homopycnic layer. To investigate the range between an N_2 and the N_1 above it, we use as a basis the possibilities 1a, 1b (N_1) and 2a, 2b (N_2) of Fig. 15. The transition from 2a to 1a yields no homopycnic layer, that from 2b to 1b, two such layers. At the transitions from 2a to 1b and from 2b to 1a, the cogent result is another H_ϱ, that is to say, we finally obtain the transition from 2b to 1b.

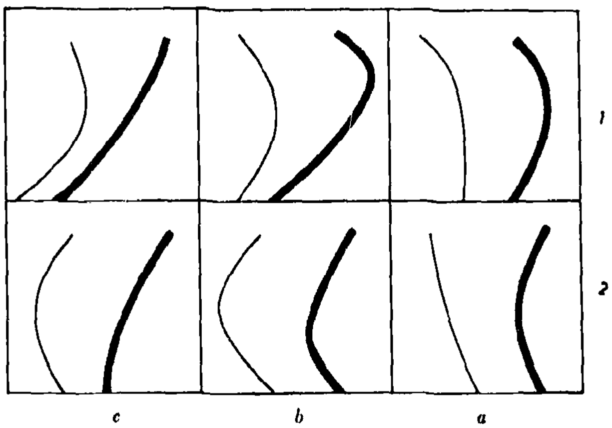


Fig. 15: Extremes of the pressure gradient (thin curves) and the wind (heavy curves).

Formally, there are also three possible connections between the minima of the zonal wind and those of the meridional pressure gradient. Fig. 15 gives a view of them. Besides the possibility of two minima appearing jointly (2b), there is one each of a minimum of only one of the two quantities occurring (2a, 2c). With respect to possibility 2b, chapter 2 permits the following statement:

C. 50 **Between a Null layer of 2nd kind and the Null layer of 1st kind below it, no homopycnic layer can develop. The vertical range between an N_2 and the N_1 above it includes either two H_ϱ's or none.**

Results attained to this point permit to represent the vertical sequence of sets between two Null layers of 1st kind (Fig. 16). For the sake of simplicity the N_1 that is

above the N_2 is supposed to be the Null layer of 1st kind that is directly above the upper boundary of the ozone layer. The upper system thus does not "oscillate" downwards. In ascertaining the relative position of a layer where the vertical temperature gradient changes its sign, the following must be taken into account. From the definition of the sets it results that as against the sets Ia, II, III, the occurrence of set II* is dependent on a positive $\partial T / \partial z$, while set Ib on a negative one. A change of sign of $\partial T / \partial z$ therefore can occur only within the ranges pertaining to Ia, II, III. Fig. 16a assumes the change of sign of the vertical temperature gradient ($T(z)_{min}$) to occur above the N_1. There are only two homothermic layers as separating layers between the lower N_1 and the N_2. Since between the N_2 and the upper N_1 there are either two homopycnic layers or none, these are shown by dotted lines, and set III which is embraced by them, is in parentheses. While in the event of the absence of the homopycnic layers both the heating level ($T(z)_{max}$) and the minimum of the vertical temperature distribution occur within one and the same set (II), the existence of set III renders four possible positions of $T(z)_{max}$ and $T(z)_{min}$ respectively. These are indicated by vertical arrows.

On Fig. 16b $T(z)_{min}$ is assumed to occur below N_2, which places the H_T above the N_2 to which it belongs, and makes it represent here the first layer of those separating the sets. Here, too, the heating level ($T(z)_{max}$) can be within the range of validity of set II, or, with the existence of set III, in the vertical range of the latter. $T(z)_{min}$ can occur only within the range of Ia.

Attention is drawn to the fact that we have intentionally plotted the lines of Fig. 16 at equal distances from one another, since no statement is possible regarding the vertical extension of the sets.

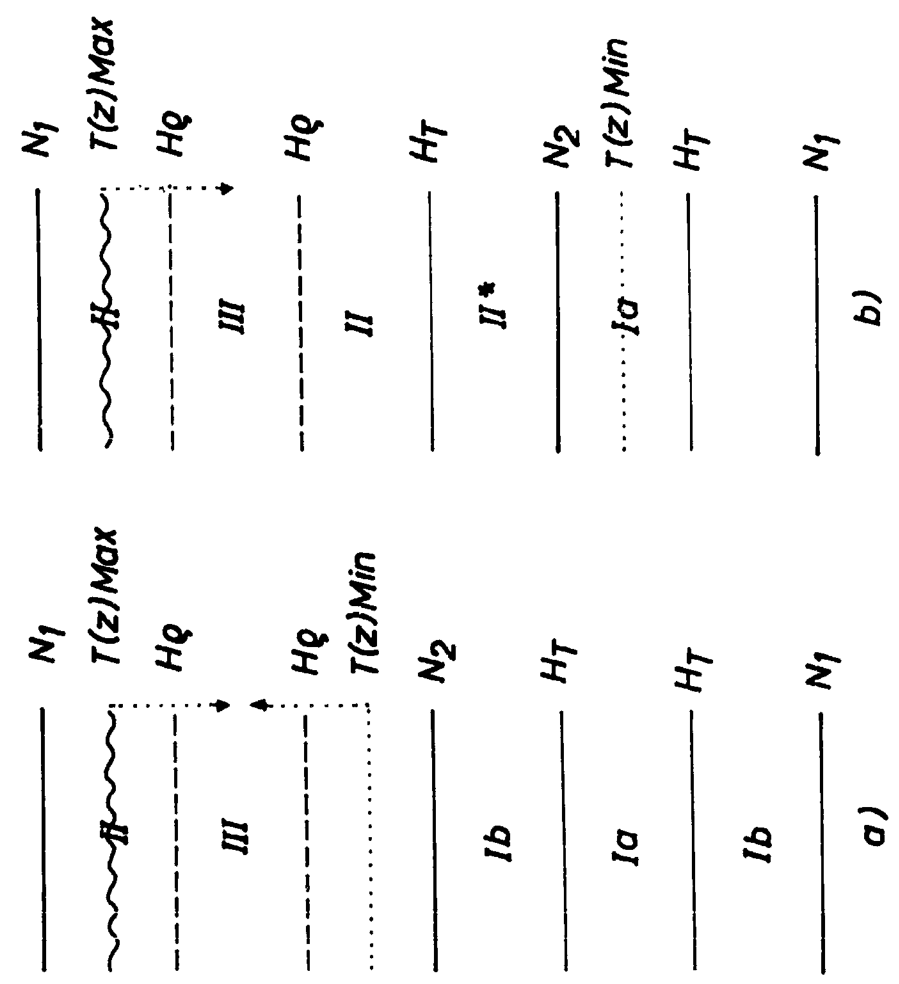
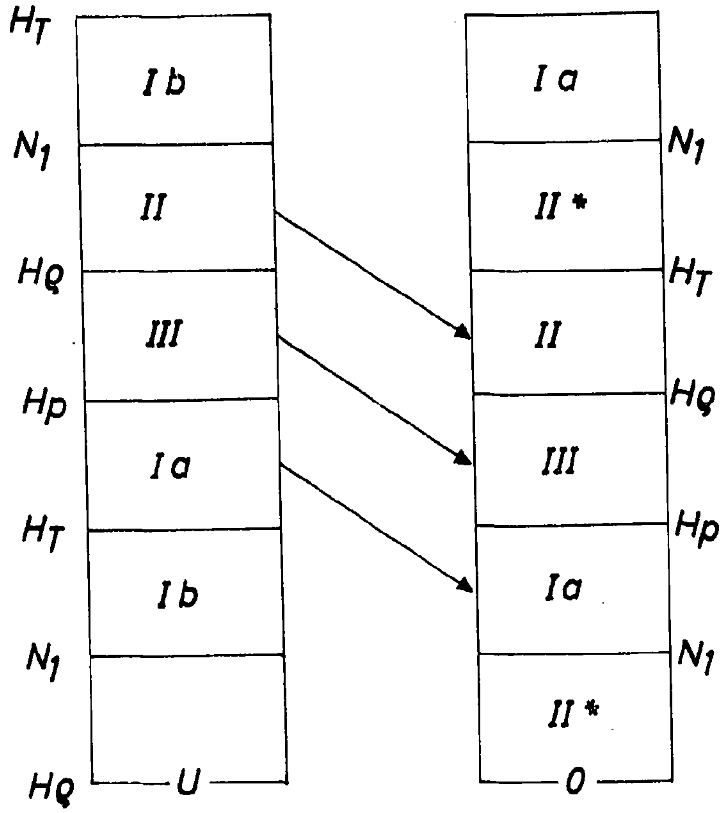


Fig. 16: Sequences of sets between two N_1



[redacted]

Fig. 17: Possible concurrent connexions between two „swinging" heating systems

Chapter 20: Manifestations of the Concurrent Transition

The definition of concurrence indicates that the vertical sequence of the marked dynamic layers of a combined system is the same as that of a single system. However, it does not say anything regarding the vertical sequence of the sets, and this will now be considered by analogy with the foregoing chapter.

Use is made of undisturbedly "swinging" lower and upper systems. Of the lower system, the part that is required to be considered here is shown at the left hand of Fig. 17, of the upper system, it is shown at the right hand. From a physical point of view it is of course impossible to regard the two systems as existing side by side; there must be a point where a passage occurs from the lower to the upper system, and below which the upper system becomes imaginary, while above it, the lower system. Thus Fig. 17 must be understood in such a way that we ascend in the lower system (left hand) up to a point which is consistent with the definition of concurrence, where we pass to the analogous point of the upper system (right hand) and proceed upwards within this. The respective arrows on Fig. 17 join the corresponding points of the two systems at which the meteorologic field parameters have the same value, thus representing the conditions in the atmosphere at a specific height. Since in the upper system $\partial T / \partial z > 0$, whereas in the lower system $\partial T / \partial z < 0$, the vertical temperature gradient must be zero at this height. That is to say, the arrows of Fig. 17 represent also points of $T(z)_{min}$, and in accordance with chapter 19 can connect only sets of the same kind with one another (Ia, II, III). Also, the connections hold for the layers separating the sets, H_ϱ and H_p, since they separate sets that exist in either system. However, the N_Is and H_Ts must be excepted for the reason that they are next to the respective sets that do not occur in the other system, and there are other reasons to except them: Prerequisite to an N_I are barotropic conditions; as $\partial p / \partial z$ cannot be zero in it, $\partial T / \partial z$, too, must be different from zero according to chapter 2. Nor can $\partial T / \partial z$ vanish in the H_T, for if it did so, the supposition as to the change of the wind with height would not be met.

The way the figure is made up does not render an explicit view of the range of transition with its particular features. In the formal transitions considered here, it is of no account whether a set has been originated by the effects of either one system or both systems. This, however, is not a full view. The joint action of both systems in the transition range may cause in it sequences of sets as do not occur in a single system. Since the combination types are dominant in the transition range, the point in question can only be changes in the sequence of the sets within one combination type. On the increase of wind with height can occur the sets II, III and II*, on the decrease of wind with height, the sets Ia and Ib. The sequence Ib — Ia — Ib cannot materialize since Ib does not occur in the upper system. Unless there is an N_I above II*, its lower boundary as well as itself will disappear. This means that above II*, there cannot be another set that includes increasing wind with height, and the sequence II — II* — II therefore is not possible. Set II, however, is retained when due to the effects of the upper system, it passes via an H_ϱ-layer to set III of the upper system.

Summarizing, we can say:

C. 51 **When the combined heating systems are concurrent, the vertical sequence of sets formally can be regarded as a transition within the range of a set belonging to both systems, from the „swinging" system with a negative $\partial T / \partial z$ to that with a positive $\partial T / \partial z$. At this point of transition, $\partial T / \partial z$ is equal to zero. An exception only is the sequence II — III — II, which may occur due to the joint action of both systems in the transition range.**

The foregoing does not affect the statements of chapter 18 concerning the separating layer, as genetic causes are not involved.

Chapter 21: Importance of the Meridional Temperature Gradients in the Heating Levels for the Vertical Structure of the Combined System

a) Summer

It is one of the basic facts in summer the meridional temperature gradient of the upper heating level is opposite to that of the lower heating level. Consistently, the directions of the wind in the vicinities of the heating levels are opposite, there is west wind above the earth's surface as against east wind at the upper boundary of the ozone layer. It has been shown that outside the transition range, either system can "oscillate". First, the simplest manifestation of the combined system is regarded. It does not include a system-owned change complicated structure. For their detailed structure, see the „model clock" in the next chapter.

b) Winter

In winter the meridional temperature gradient of the upper heating level shows in the same direction as that on the earth's surface, that is, southward. Under these conditions it is the countercurrent combination system which is the simplest (Fig. 21). As for the simplest concurrent combination system, this results either in the case of each system producing one system-owned H_p,

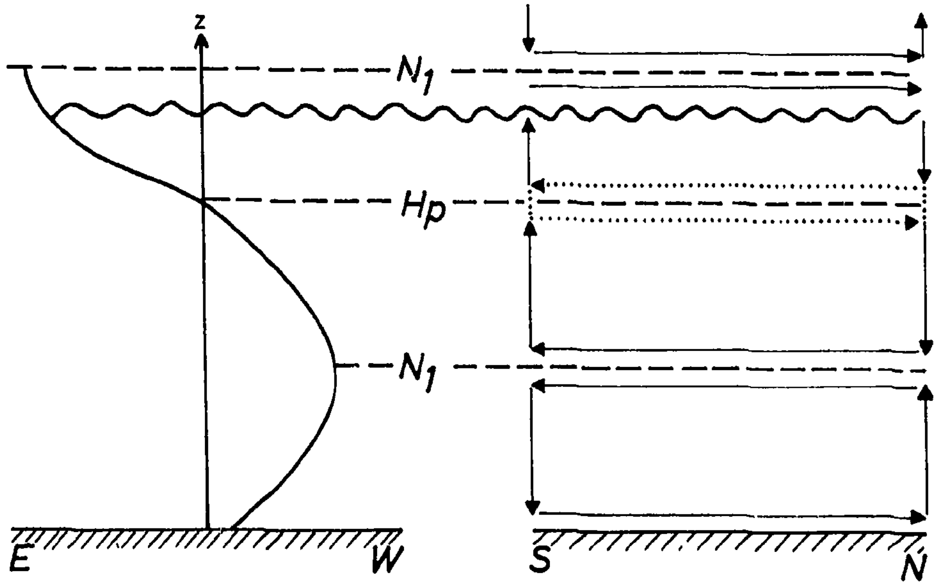


Fig. 18: The simplest concurrently combined system in summer (waving line = heating level)

of the wind direction; this corresponds to the simplest case of concurrence, with a minimum number of marked dynamic layers (viz. 2) between the heating levels. Fig. 18 illustrates the behavior of the wind field and the resulting circulation scheme in this case.

We now consider the combined systems with one system-owned H_p. In summer this represents the simplest case of countercurrence, though it is not unambiguous any longer because the H_p may occur either in the lower system (Fig. 19a) or in the upper system (Fig. 19b).

Theoretically it is possible that both the systems develop one or more system-owned H_p's, this resulting in concurrent or countercurrent combination systems of a or in the case of one system producing two such layers. Schematically the vertical wind profiles are the same with any of these three possibilities (Fig. 20).

The complicated combination systems which are possible theoretically can be found also by means of the model clock.

*

It does not cause any difficulties in neither of the two seasons to determine the separating layer of a countercurrent combination system; this cannot be another layer but the Null layer of second kind. But with the concurrent combination system the separating layer can only be found using the definition given in chapter 18.

As yet merely the direction of the meridional temperature gradient in the heating level has been taken into account, and has been sufficient to show the possible forms of occurrence of the combination systems, for the reason that both the terms of concurrent and countercurrent are independent of the intensity of the heating level. The values of the meridional temperature gradients in the heating levels have so far been ignored for

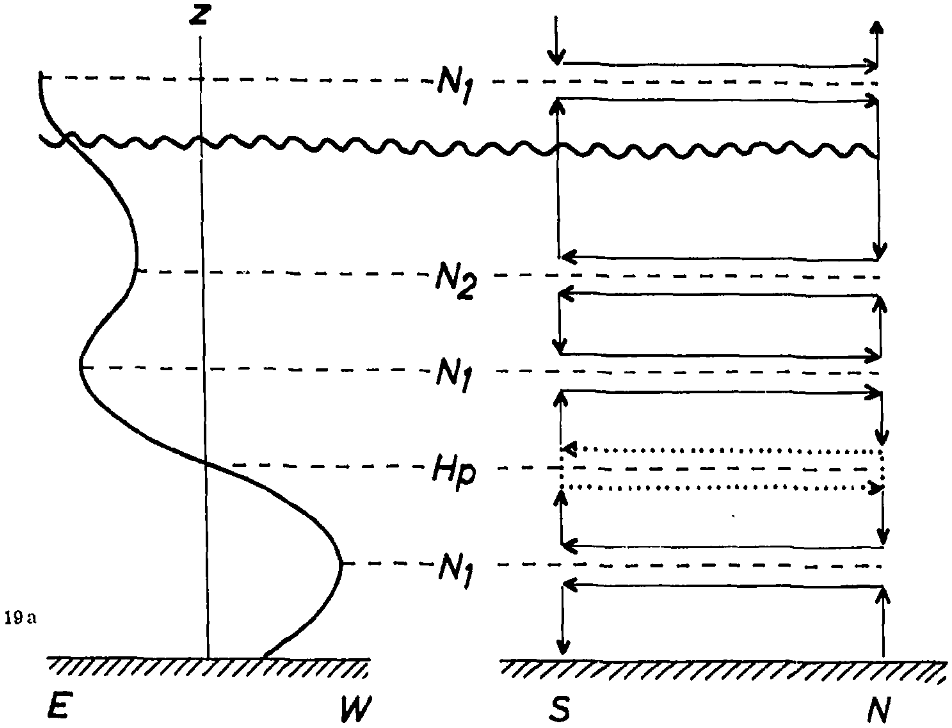
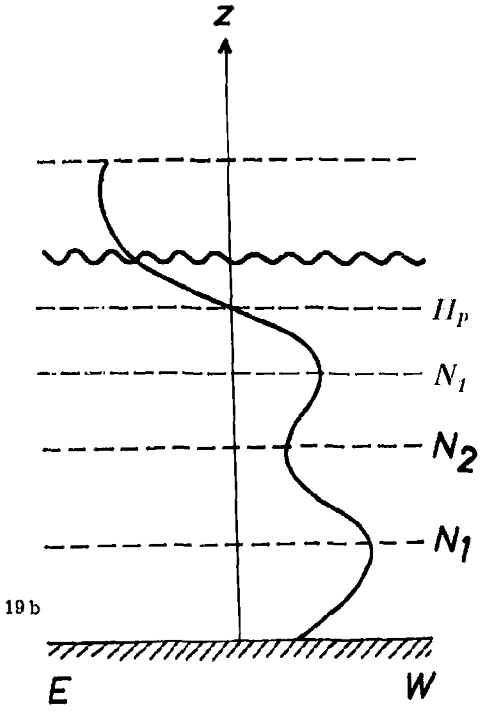


Fig. 19: The simplest countercurrently combined system in summer



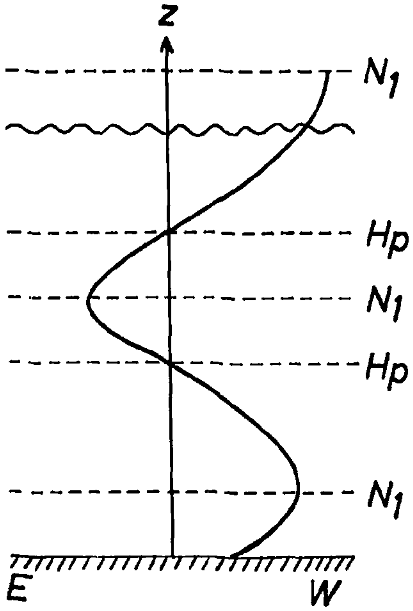


Fig. 20: The simplest concurrently combined system in winter

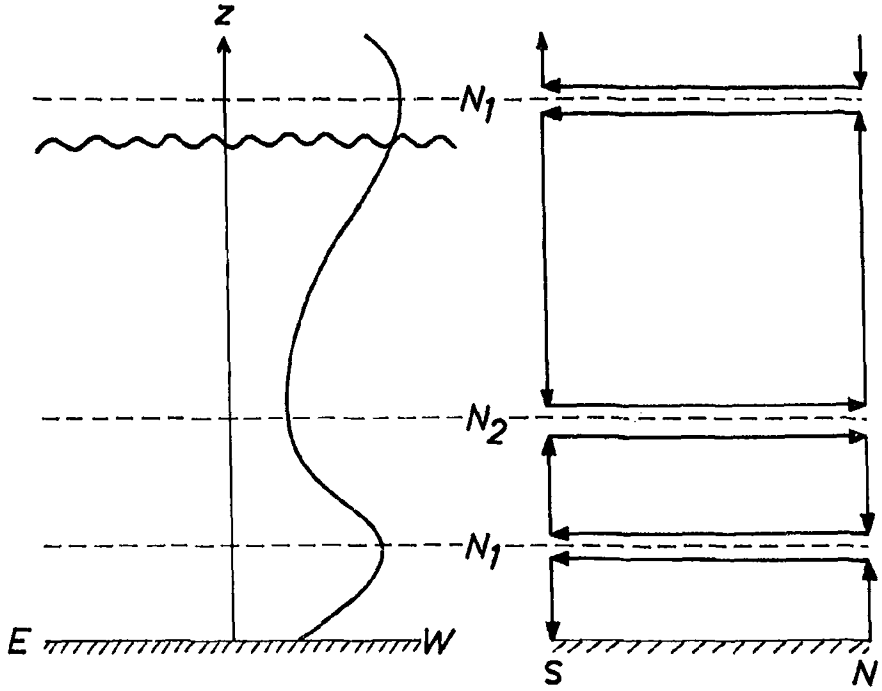


Fig. 21: The simplest countercurrently combined system in winter

the difficulty of determining their effects, on the basis of the results found here. For instance, there still remains the question whether an increase in the value of the meridional temperature gradient in the heating level causes an increase, or a decrease, of the respective vertical distances between the marked dynamic layers, and what effects this may have upon the range of validity of the respective heating system. Relation between the mean heating ($\partial T_m / \partial z$) and the mean temperature difference in the heating level ($\partial T / \partial y$) may be essential in this respect. These problems are not just academic but may prove to be of considerable importance regarding the temporary changes of the radiative conditions (e. g. the seasons).

This brings about considerations of a causal, physical nature concerning the mechanism of interaction of two heating systems. The strictly formal access which has been used here in order to offer a generous view of the vertical structure of a combination system, has included two mutually independent heating systems together with a well-defined transition range between them. This it has been legitimate to do when making qualitative considerations. Quantitative analyses, however, will not allow to maintain such a restriction, but will rather require the mutual influences to be taken into account which occur in the full range of the heating systems or at least in a large part of them.

The concluding chapter will refer to other open problems as well as to ways how to solve them. Preceded will this be, however, by a chapter to enable the deduction of the vertical structure of an atmosphere including any number of heating levels.

Chapter 22: The Model Clock - Diagrammatic Representation of the Above Results to Permit to Determine the Vertical Sequence of Sets in an Atmosphere Comprising Heating Levels

Our aim in the foregoing chapters has been to elucidate the joint action of two heating levels, selecting from the variety of their possible combinations the simplest one. In order to be able to embrace the whole of the variety of the conditions caused by two heating levels, a graphic means of representation, which we call model clock, has been devised. This quasi-cyclical representation permits to determine even the fine structure of an atmosphere including any number of heating levels. However, it has been shown in the above chapter that the quantitative relationships cannot be surveyed yet, and while the model clock enables all the possible kinds of vertical structure to be determined, no method has up to this been deduced to determine that particular kind which would correspond to given basic quantities. In other words, it cannot be said whether, and how often, a system "oscillates". As basic quantities we may regard the respective meridional temperature gradients in the heating levels, the amount of their heating ($\partial T_m / \partial z$) and their mutual distance.

On dealing with the conditions above one heating level ($\partial T / \partial z < 0$) the physical necessity of the sets occurring side by side as shown in chapter 6, has reappeared in chapter 12 in the form of a systematic, vertical sequence. Within a „swinging" atmosphere additionally a cycle is represented by the sets III — II — Ib — Ia, and it is obvious to illustrate their vertical sequence as a circle (Fig. 22). It is for symmetry among other reasons that here, too, the sets are shown at equal vertical extensions. The marked dynamic layers, viz. the Null layer and the homobaric layer, divide this circle vertically, and through a v_z-bound and a v_x-bound, into two half circles with increasing (left hand) and decreasing (right hand) zonal winds respectively. The semicircles are again divided by the other set-separating layers, H_T and H_Q. The abbreviations of the sets, which are now localized, are given within the respective quadrants. The vertical structure of a "swinging" heating system can be found from Fig. 22 by following the dial clockwise, twice in respect of each "oscillation"; however, on condition that

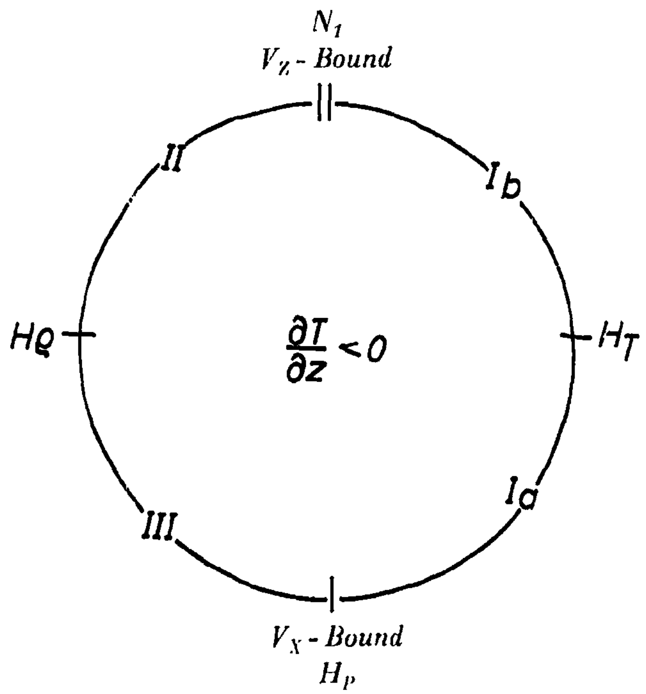


Fig. 22: The model clock
The lower „swinging" system

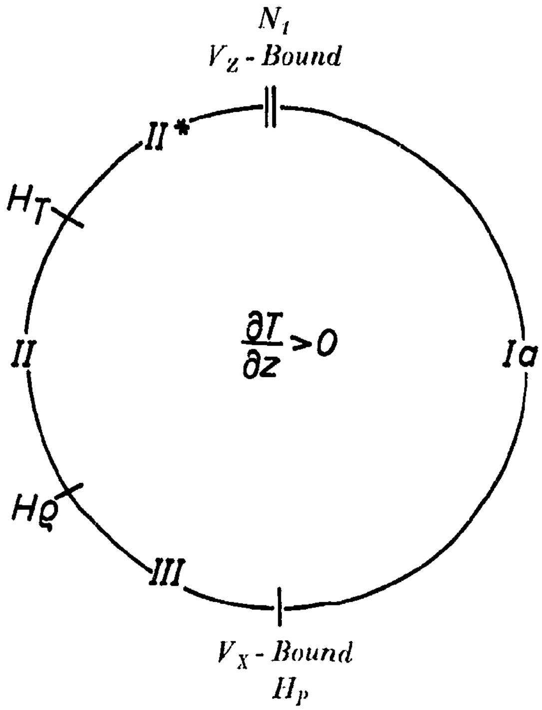


Fig. 23: The model clock
The „upper" swinging system

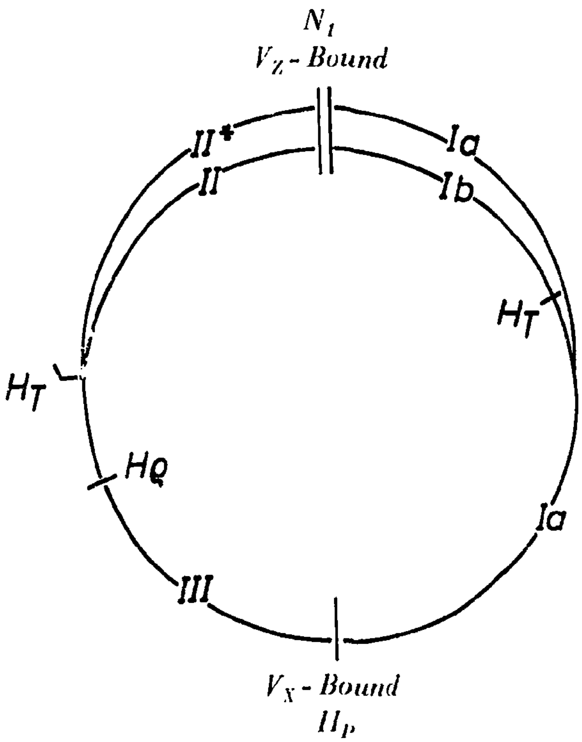


Fig. 24: The model clock
Combination of figures 22 and 23

the directions of both the zonal wind and the vertical motion are given in the starting point.

A dial for $\partial T/\partial z > 0$ by analogy with the above is Fig. 23. With this, the semicircles cannot be symmetrical for the reason that now the range of increasing zonal wind covers three sets while that of decreasing zonal wind only one.

Fig. 24 is a fusion of the two dials, allowing to deduce the condition of the atmosphere except for the range of transition. For $\partial T/\partial z$ negative, use the inner band, for $\partial T/\partial z$ positive, the outer. The H_T, which separates set II from II*, is of importance only for the latter band.

In view of the rather complicated possibilities included in the transition range we shall continue to construct the model clock step by step, first illustrating the manifestations of the concurrent transition referred to in chapter 20 (Fig. 25). Here, a $\partial T/\partial z = 0$ layer can occur within the sets Ia, II, III, and because of this variety, the respective place has not been marked on the model clock. In accordance with conclusion 51, Fig. 25 ist a supplement to Fig. 24 in that it considers the sequence of sets II — III — II.

The model clock is to be used thus: Follow the track of the lower system ($\partial T/\partial z < 0$) up to a given place where $\partial T/\partial z = 0$; then, with the alternative of using the byway (III), proceed around the outer band where $\partial T/\partial z > 0$, up to the next heating level ($\partial T/\partial z = 0$). As shown earlier, it must be situated either in set II or III. Since the heating system we are in is "swinging", the byway (III) is not applicable. $\partial T/\partial z$ is negative above the heating level, which again requires the inner band to be used.

Two set sequences are mainly supplied in the transition range by the countercurrent transition (chapter 19), according as to whether $\partial T/\partial z = 0$ occurs below or above the Null layer of 2nd kind. Considering at last these two sequences renders the final shape of the model clock (Fig. 26).

If $\partial T/\partial z = 0$ occurs above the N_2, pass from set Ia of the lower system to the dashed line, along which to reach set II of the undisturbedly "swinging" upper system. Here the byway (III) cannot be excluded either. In the case of $\partial T/\partial z$ reversing below the N_2, leave the lower system at set Ia, and proceed along the dash-dotted line and around the circuit of the upper system. The alternative of the byway (III) cannot be used unless in connection with the transition.

When using the model clock it must be remembered that the definition of the sets does not provide an interpretation of the directions of the zonal wind and the vertical motion, and the conditions at a specific point of the atmosphere therefore are not fully explained by just

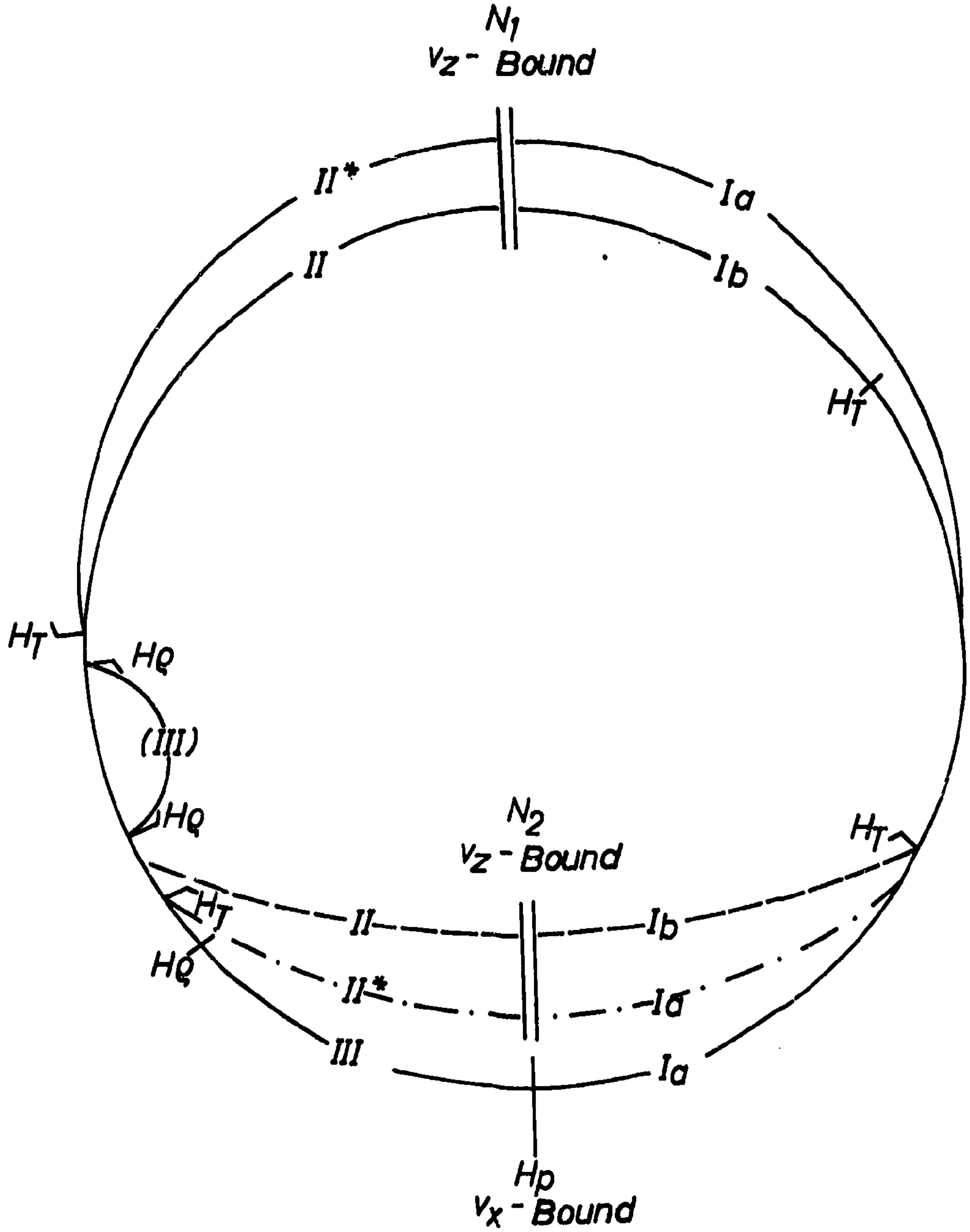


Fig. 26 The complete model Clock

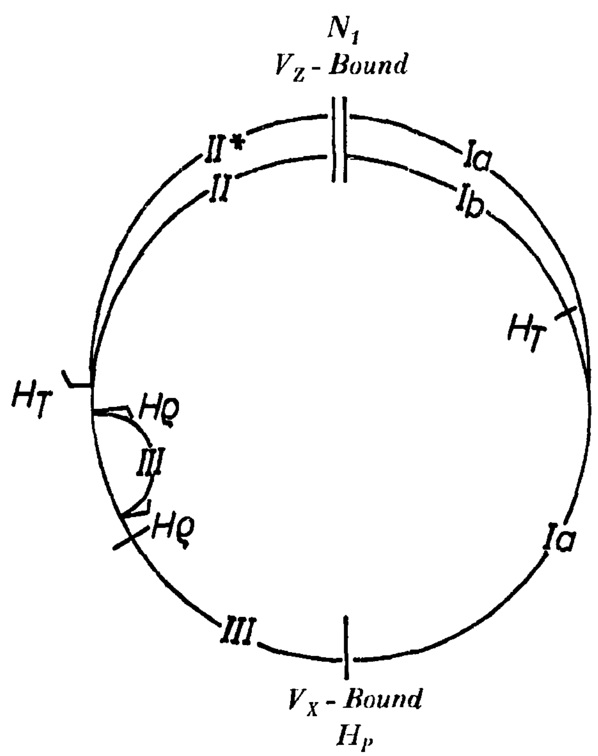


Fig. 25: The model clock
Consideration of the concurrent transitions

naming the respective set. Mean conditions require the directions of v_x or v_z to be stated as well as what branch of the circulation wheel is being dealt with. For according to chapter 18, it is only certain assignments that are possible in respect of v_x and v_z at the high respectively the low latitudes. The basic facts furnish a definition of $\partial p / \partial y$ in the heating levels, and accordingly, of the direction of the wind. When inserting these quantities in the model clock at a specific branch of the circulation wheel, the directions of both v_x and v_z are automatically rendered for all sets, since N_1 as well as N_2 reverse the vertical motion, and H_p reverses the zonal wind.

The model clock serves to represent most of the essential results obtained in this analysis. It does not include the pleisto-layers, which do not own set separating characteristics, and consistently, does not illustrate their relative positions to the other layers. However, there must be at least one pleisto-layer between two layers of vanishing meridional gradients of a meteorologic parameter. Hence the model clock concludes to a certain extent the theoretical investigations.

V Concluding Remarks

Chapter 23: Brief Comparison with Meteorological Experience

A scrutiny of this model in the light of empirical results will itself yield a comprehensive paper, and is to be supplied in a separate Scientific Report. The object of this section is to offer a brief comment.

In comparing the above-derived layers with the conditions in nature a disadvantage is the lack of absolute mean altitudes. But even without these, the layers deduced here are easily identified with those of which the knowledge is empirical.

The deduced homopycnic layer is (over the middle latitudes) situated at a mean height of 8 km [1, 14].

The Null layer of 1st kind is situated at a mean height of 10 km [7, 15, 16], its existence being proved by the persistency of the high-tropospheric wind maximum. The wind maximum would have to be disturbed within a short period by friction, turbulence and the interchange of air masses, and as shown by B. Haurwitz [17] the winds in it would have to be subgradientic and to have a flow component towards the low pressure. It is, however, known from experience that the wind maximum is persistent and, whenever dissolved, reappears after a two or three days' period. This furnishes the proof that the effects maintaining and rebuilding the wind maximum are predominant over those trying to destroy it. The only conceivable mechanism which can be responsible for the maintenance of the wind maximum is given by the vertical motion system being deduced in chapter 4. Thus the empirical persistency of the high-tropospheric wind maximum is the simplest, and best, proof that this wind maximum layer owns Null layer characteristics. The same holds true analogously for the other layers of extreme winds deduced here.

Conclusion 5 says that the homothermic layer belonging to the N_1 is situated "slightly above the Null layer". This homothermic layer, which is connected with a reversal of the meridional temperature gradient, is apparent from [14, p. 201, 15], and especially from the Scientific Report No. 8 of the Project's third year (1961/1962), which offers a diagrammatic view of the mean p, T and ϱ-differences between Fairbanks (64° N, 147°W) and Patrick (28° N, 80° W) [18]. Empirical investigations so far carried through have not yet yielded a clear proof as to whether or not the homothermic layer is indeed situated "slightly above the Null layer". The evaluation of partial results would indicate the actual difference in height between these two layers to be of the right sign.

[18] also proves the existence of the deduced layers of maximum meridional gradient of pressure, which coincide with homopycnic layers, as well as of those of temperature and density. They are apparent from [15, 19] also. Due to advection, a maximum of the local dispersion is caused by them at that specific height. Such a vertical distribution of density dispersion is shown i. g. in [20], and in fact proves the findings. Corresponding investigations have been started with respect to the vertical distribution of temperature as well as of pressure dispersion. The empirical results available reveal that in the yearly mean for the temperate latitudes, the pleistopycnic layer above the N_1 is situated at 12 km, and the pleistothermic layer above the N_1 at 14 km. There are only weak indications in [18] (system high latitudes — low latitudes) of the occurrence of the deduced pleistothermic layer below the Null layer, while there are none at all of the occurrence of the respective pleistopycnic layer.

In the high/low system the pleistothermic layer is well-known from synoptic experience. It is situated at about 5 km. The authors have not so far met with any empirical data indicating the behavior of the horizontal density gradient at those altitudes (high/low system), and examinations to furnish an empirical proof have been started.

The Null layer of 2nd kind is known from [15, 21, 22, 23], the pertinent reversal of the meridional temperature gradient from [15, 16, 18, 21]. The N_2 occurs only in winter, its mean altitude being 20 km. In summer, there is at that altitude a homobaric layer connected with a reversal of the mean zonal wind direction [15, 16, 18, 21].

The existence of a homopycnic layer above approximately 20 km had not been clear until recently. When in setting up this model, its existence was theoretically found for the summer though not for the winter, a comprehensive statistical approach was made [18] to the problem, which proved the theoretic results to be correct. The findings are also confirmed by the recent publication [24].

Unlike the foregoing conditions, those above 30 km of course are not so easily to be proved empirically. Never-

theless a persistent wind maximum with a reversal of the horizontal temperature gradient has been found at about 55 to 60 km by W. Attmannspacher [15, 25], which displays west winds in winter and east winds in summer. This represents the N_1 deduced to occur above the upper boundary of the ozone layer (chapter 14). Using the altitudes known to involve the various Null layers, H. Faust (1957) constructed a mean general circulation scheme reaching up to 60 km [26], which he later on completed with the cooperation of W. Attmannspacher [27]. By its vertical motion, this circulation scheme interprets easily the peculiar temperature data collected on occasion of the Rocket Grenade Experiment [28, 29, 30] at a height of 80 km over Fort Churchill, Canada, where temperatures in winter are more than 60° C above those in summer. More recently H. Faust [31], using the Rocket Grenade Experiment temperature data, showed by aid of the circulation scheme the relative temperature distribution which has to be expected between the high and the low latitudes at these heights. A proof of this was furnished by W. Attmannspacher and J. Tòth [32], when a number of rocket data were available from the low latitudes. The reason for calling attention to the proof of the abovementioned circulation scheme is that this scheme was derived directly from the respective positions of the Null layers, which arrangement has reappeared in this theoretical analysis.

Regarding the meridional density gradient of the upper heating level, no unambiguous empirical data are known to the authors at this point. E. Soòs, within the framework of the Project and using the available rocket data, has commenced to examine those findings from an empirical point of view.

The foregoing coincidences with nature of the results of this model have been given for examples. Generally speaking, the authors are not aware of the existence of any findings cogently contradictory to the model deduced by them.

There is another point that needs attention. While Part IV has dealt with the possible occurrence of additional "oscillations" of either of the two heating systems, it has not decided whether similar „oscillations" occur in the mean in combined systems or not. To answer this question the same amount of complicated calculations would have been necessary as to find the absolute heights of the individual layers deduced here. The determination of mean conditions thus have been left to empirical evaluation. This indicates [e. g. 33] that in winter there is mean west wind from the surface up to above 60 km (N_1); in summer, the west wind shifts to east at 20 km, to keep this direction up to above 60 km (N_1). That is to say, *additional oscillation does not occur in the mean in either of the two systems.* The separating layer in the mean therefore is dependent upon the directions of the respective temperature gradients in the two heating levels: In winter, these are parallel to one another, with the two heating systems linked countercurrently (N_2), in summer, they are antiparallel, with the two heating systems linked by concurrence (H_p).

It is to be shown by empirical data whether additional oscillations of a heating system — perhaps limited in space — may occur in particular cases.

Chapter 24: Conclusions and Further Problems

Dispensing with the use of empirical knowledge, the authors' aim in this analysis has been to ascertain such marked characteristics of the atmosphere as are based on very simple suppositions. The major suppositions of those not utilized are the consequences of the existence of water vapour in the atmosphere and the presence of the tropopause. Considering the important rôle of water vapour in the tropospheric weather, this result becomes worthy of note. In the case of the findings of this analysis being consistent with the conditions in nature, and there is no indication as yet that they are not, these mentioned facts will not, therefore, play a decisive part.

For its feature of separating two atmospheric regions the tropopause is occasionally named one of the most important boundaries of atmospheric levels. From the above study it must be concluded, that this level cannot enact an important part in dynamical processes. On the contrary W. Attmannspacher [34] has shown that hitherto inexplicable characteristics of the tropopause (e. g. types, vertical dislocations) can be explained by aid of the Null layer conception.

Instead of to the tropopause, this paper attaches fundamental importance to the separating layer (at about 20 km). This layer is represented in summer by an H_p and in winter by an N_2.

The justification of putting such an emphasis in this study upon the vertical motions is that the layers deduced through them are in fact found in nature. The vertical motions are connected with nongradientic winds, whereas the calculations in this paper in what might appear to be a contradictory way have been effected gradientically. However, the calculations have been led only so far as to permit to draw physically simple, but cogent, conclusions. Then the nongradientic flow of air masses toward the high pressure in the N_1 has been found simply, and physically cogently, from the principle of continuity, avoiding complicated nongradientic calculations. The nongradientic flow of air masses deduced by aid of the principle of continuity is consistent with the vertical distribution of both convergence and divergence of the adiabatically deduced vertical motions, which again are based on the vector product of two gradients of the parameters p, T and ϱ.

In a theoretic work which he has not yet published, W. Attmannspacher has found several terms for the vertical motion, one of which, and apparently the predominant, is the abovementioned vector product. This is therefore the only one which has been utilized in this analysis, the subsequent comparison of the findings with nature having proved this to be correct. As far as the actual state of knowledge indicates, the vector product offers a reasonable description of the large-scale vertical motion of the atmosphere.

The previous chapter shows Fig. 18 and 21 to represent the mean circulation scheme for both summer resp. winter (no additional "oscillation"); the empirical absolute heights given above enable the km figures to be included in it. It is, however, known from experience [35] that especially in winter, another circulation wheel with direct thermal circulation (descending cold air) is included extending from the earth's surface up to about 3 km, in the range of the flat polar cold air. This polar high-pressure system has been derived in chapter 10 without calling attention to the fact.

The circulation schemes themselves have been deduced assuming that at any fixed elevation the meridional gradients do not change their signs in proceeding along the meridian. However, as known from experience there may in single cases prevail high pressure, at high latitudes, within the lower dynamic system (0 to 20 km). Similar large-scale deviations from the mean conditions will have to be investigated later on in connection with a division of the mean circulation wheel into individual wheels covering the meridians.

Energetic questions have not been raised, and it seems advisable therefore to throw some light upon the problem of the transference of energy toward the N_1. Energy is involved in the nongradientic flow of air masses toward the high pressure. G. Hollmann [36] revealed that the ascending air of the low-pressure system below the N_1 causes more kinetic energy of the horizontal air motion to be transferred toward the N_1 than downward from it in the high-pressure system which owns less energy. At the N_1 consequently takes place an accummulation of kinetic energy, which is consumed by the work performed by the flow of air masses toward the high pressure occurring in conjunction with supergradientic winds. Reciprocally kinetic energy becomes with-drawn from both sides of an N_2.

The development of an analogical homopycnic layer seems to be a function of the specific weather situation (bottom set II or III). The occurrence of the bottom sets Ia, Ib, II and III in connection with the development of the pressure systems is supported by various considerations, and is to be investigated, since it will be a matter of considerable consequence. Equally, an empirical reply is to be given to the question of the development

of a P_ϱ layer at 12 km in the case of the H_ϱ layer below the N_1 missing (bottom set II).

Except in a few cases, it are only pleisto-layers and homo-layers that have been dealt with in this analysis. Below and above the 20 km H_p, pleistothermic layers whose meridional gradients have the same direction would, however, be expected to occur due to the like-signed vertical motions. Consequently, there would have to be a minimum condition of the meridional temperature gradient in the vicinity of the H_p. It is felt that such layers of a real minimum condition of the meridional temperature gradient, which have not been given a name, should also be examined.

The way the heating levels have been used would have required the emission levels to be included. But these will not influence the direction of the circulation unless they reverse the meridional temperature gradient deduced above, and such type emission levels are not known as yet.

The analysis cannot simply be carried to higher levels due to the really complicate effects of the magnetic field of the earth upon the ionized air particles as well as of the tides [37, 38].

Of a much more complicated kind are the circulative conditions in the seasons of both spring and fall [39]. Although appropriate considerations have been made which have shown additional, vertically moving Null layers to occur [40], a separate report is to be written regarding them so as to avoid this publication becoming too voluminous. Equally, a model of the tropical latitudes, in respect of which a direct thermal circulation is assumed to exist, is to be dealt with at some future time. There, too, the persistent wind maxima [41] seem to possess Null layer characteristics, the flow of air masses, however, taking place presumably from the high pressure to the low [42]. [15, 43] make this likely in a Null layer of 1st kind for the cases of weakening highs and lows. The circulation in such weakening pressure systems indeed is considered to take place in a direct thermal way.

Of course the value of this analysis (and of those concerning the spring and fall, and the tropics) is not restricted on theoretical considerations. First the results are to be based on empirical data, when the practical requirements may be met as far as reasonable, viz. by embodying the average values of the meteorologic parameters at the individual layers and latitudes, the values of the meridional gradients, the dispersion, the frequency distribution etc. Figures may be given for the individual months, the respective large-range weather conditions, and for both the land and sea stations. In such work the model may serve as a frame. However, the labor involved will be so extensive as to require electronic computers to be employed.

An important practical application of the model is in connection with the variation of the solar ultraviolet rays. The radiant energy given to the second heating system (20 to approx. 80 km) appears to be intensified at one time while reduced at another. The variation of ultraviolet energy measured by OSO satellites is to be compared with the behavior of the upper system [44] observed on rocket firings. Additional "oscillations" of either of the two dynamic systems may be found to occur in certain cases. But the major question to be clarified is what effects any changes of the energy of the upper system may have upon the behavior of the lower system. The influences exerted by such processes on the weather conditions have been revealed by the large-range weather research, particularly by the pioneer work of F. Baur [e. g. 45]. The physical processes underlying such a vertical coupling of the two systems are to be subjected to thorough investigation. The various assumptions that have been made in this respect could not be maintained, mainly because of the reduced air density at high altitudes. This model has been found to offer a noncontradictory suggestion how to imagine physically the coupling of the processes of the two systems to take place. For its hypothetical nature, this suggestion has, however, not been included in this report. It has been analyzed by W. Attmannspacher in [46]. Further empirical work in respect of those problems on the basis of this model are expected to contribute largely to the large-range weather research as well as to its practical application, the long-range and middle-range forecasts.

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Literature

[1] Linke, F.: Über die Luftdichte I. Beitr. z. Phys. d. fr. Atm. **8**, 73—85 (1919)

[2] Wagner, A.: Die Temperaturverhältnisse in der freien Atmosphäre. Beitr. z. Phys. d. fr. Atm. **3**, 57—167 (1910)

[3] Linke, F.: Über die Luftdichte II. Beitr. z. Phys. d. fr. Atm. **8**, 194—199 (1919)

[4] Doporto, M.: The Computation of Atmospheric Pressure at the 8 km Level of Constant Air Density. Dep. Ind. a. Comm., Met. Service, Geophys. Publ., **III**, **4** (1951)

[5] Attmannspacher, W.: Extreme der horizontalen Windgeschwindigkeit und Vertikalwind. Met. Rdsch. **12**, 112—117 (1959)

[6] Hollmann, G.: Zur Frage des Vorzeichenwechsels der Vertikalbewegung beiderseits isobarer Flächen extremer Windgeschwindigkeit. Met. Rdsch. **12**, 137—138 (1959)

[7] Faust, H.: Die Nullschicht, der Sitz des hochtroposphärischen Windmaximums. Met. Rdsch. **6**, 6—15 (1953)

[8] Faust, H.: Wind- und Luftdruckverhältnisse bei veränderter Rotationsgeschwindigkeit der Erde. Met. Rdsch. **1**, 97—99 (1947)

[9] Lettau, H.: Über maximale Gegensätze des Luftdruckes als Funktion der Erdrotation. Met. Rdsch. **1**, 99—100 (1947)

[10] Lettau, H.: Maximalwerte der Windgeschwindigkeit als Funktion der Erdrotation. Met. Rdsch. **1**, 451—453 (1948)

[11] Wippermann, F.: Maximale Luftdruckgegensätze und Windgeschwindigkeiten als Funktion der Erdrotation. Met. Rdsch. **2**, 257—258 (1949)

[12] Dahler, H.: Die Bedeutung der Temperaturadvektion für die Bildung und Verlagerung von Strahlstrom und Tropopause. Z. f. Meteorol. **13**, 205—211 (1959)

[13] Milankovitch, M.: Mathematische Klimalehre und astronomische Theorie der Klimaschwankungen. Berlin 1930

[14] Hann-Süring: Lehrbuch der Meteorologie, Band I., S. 248, Leipzig 1939

[15] Faust, H. and Attmannspacher, W.: Cell Structure of the Atmosphere. Final Report, Contract No.: Da/91/505/EUC/387 (1959)

[16] Attmannspacher, W. and Soós, E.: On Windextremes in Troposphere and Lower Stratosphere. Scientific Report No. 5, Contract No.: DA/91/591/EUC/1668/ OI /7330/61 (1961)

[17] Haurwitz, B.: Frictional Effects and the Meridional Circulation in the Mesosphere. Jour. of Geophys. Res. **66**, 2381—2391 (1961)

[18] Faust, H. and Attmannspacher, W.: Horizontal Gradients of Pressure, Temperature and Density to a Altitude of 30 km (Fairbanks-Patrick and Patrick-Albrook), Scientific Report No. 8, Contract No.: DA/91/591/EUC/1668/ OI /7330/61 (1961)

[19] Schneider, H.-D.: Temperature and Density Changes in the Troposphere and Lower Stratosphere with Coldfront Passages. 1st Special Report, Contract No.: DA/91/591/EUC/1668/ OI /7330/61 (1961)

[20] Reisig, G. and Alfuth, W.: Global Density Deviations Profiles Independent of Geographic Location and Seasons. ABMA, Aeroballistics Lab., Report No. DA-TN 31—58. 1958

[21] Faust, H.: Die Schwachwindschicht in der unteren Stratosphäre als Nullschicht. Met. Rdsch. **13**, 77—85 (1960)

[22] Freitag, E. and Faust, H.: Sub-Gradientic Winds of the Winterly Null Layer of 2nd Kind. Scientific Report No. 7, Contract No.: DA/91/591/EUC/1668/ OI /7330/ (1961)

[23] Paetzold, H.-K. and Regener, E.: Ozon in der Erdatmosphäre. Handbuch der Physik, Bd. XLVIII, Geophysik II, 1957

[24] Alfuth, W.-H., Dickey, L.-R. and Alsobrook, A.-P.: An Attempt to Establish a Standard Atmosperic Density Profile under Consideration of its Time and Space Variations. Redstone Arsenal, Alabama, Report No. RR-TR-3-60, Army Ballistic Missile Agency (1960)

[25] Attmannspacher, W.: Über die Existenz einer hochstratosphärischen Nullschicht 1. Art. Met. Rdsch. **13**, 38—45 (1960)

[26] Faust, H.: Die Abwechslung dynamisch polarer Schichten als Prinzip der Atmosphäre. Raketentechnik und Raumfahrtforschung 1960, 81—85

[27] Faust, H. and Attmannspacher, W.: Die allgemeine Zirkulation der außertropischen Breiten bis 60 km Höhe auf der Basis der Nullschichtkonzeption. Met. Rdsch. **14**, 6—10 (1961)

[28] Bandeen, W.-R., Griffith, R.-A., Nordberg, W. & Stroud, W.-G.: The Measurements of Temperatures, Densities, Pressures and Winds over Fort Churchill, Canada, by Means of the Rocket Grenade Experiment. US Army Signal Research and Development Laboratory Fort Monmouth, N. J., (1959)

[29] Stroud, W.-G., Nordberg, W., Bandeen, W.-R., Bartmann, F.-L. & Titus, P.: Rocket-Grenade Observation of Atmospheric Heating in the Arctic. Geophys. Res. **64**, 1342 (1959)

[30] Stroud, W.-G., Nordberg, W., Bandeen, W.-R., Bartmann, F.-L. & Titus, P.: Rocket Grenade Measurements of Temperatures and Winds in the Mesosphere over Ford Churchill, Canada. IAS Paper No. 60—47 (Inst. of the Aeronautical Sciences, New York 21) (1960)

[31] Faust, H.: Zu den Temperaturmeßergebnissen des „Rocket-Grenade"-Unternehmens. Geof. Pura e Appl. **47**, 183 (1960)

[32] Attmannspacher, W. and Tóth, J.: Zur Frage der Richtung des mittleren meridionalen Temperaturgradienten in der Stratosphäre und unteren Mesosphäre. (Im Druck)

[33] Attmannspacher, W.: Average Wind Conditions between the Altitudes of 14 and 30 km over North America. Scientific Report No. 6, Contract No.: DA/91/591/EUC/1668/ OI /7330/61 (1961)

[34] Attmannspacher, W.: Null Layer and Tropopause. Scientific Report No. 2, Contract No.: DA/91/591/EUC/1668/ OI /7330/61 (1961)

[35] Flohn, H.: Studien zur allgemeinen Zirkulation. Bericht Deutscher Wetterdienst, US-Zone, No. 18 (1950)

[36] Hollmann, G.: Zur Frage des Nullschichteffekts und der Strahlströme. Met. Rdsch. **7**, 166—170 (1954)

[37] Phillips, G.-H.: Measurements of Winds in the Ionosphere. J. Atm. Terr. Phys. **2**, 141 (1952)

[38] Deb, S.: Winds in the Ionospheric Regions. J. Atm. Terr. Phys. **4**, 28 (1953)

[39] Faust, H. and Attmannspacher, W.: Zur Jahreszeitenabhängigkeit der Stratosphärenwinde. Met. Rdsch. **14**, 178—180 (1961)

[40] Attmannspacher, W.: Zur Umstellung der großräumigen Zirkulation in der Stratosphäre. Geof. Pura e Appl. **49**, 178—190 (1961)

[41] Soós, E.: Windmaxima mit und ohne Nullschichtcharakter. Met. Rdsch. **12**, 118—120 (1959)

[42] Faust, H.: Meridionalgradienten und zeitliche Änderungen von Druck, Temperatur und Dichte. Met. Rdsch. **15**, 65—70 (1962)

[43] Faust, H.: Die Nullschicht bei Druckgebilden in Abschwächung. Met. Rdsch. **14**, 78—85 (1961)

[44] Faust, H.: The Significance of Radiation Measurements by Satellites for Meteorology. Technical and Scientific Report No. 3, Contract No.: DA/91/591/EUC/1295 (OI/4152/60 (1960)

[45] Baur, F.: Physikalisch-statistische Regeln als Grundlagen für Wetter- und Witterungsvorhersagen. Bd. I und II, Frankfurt/M. 1956 und 1958

[46] Attmannspacher, W.: A Hypothesis on the Influence of the Circulation Systems above and below the Separating Layer at 20 km Altitude. Scientific Report No. 9, Contract No.: DA/91/591/EUC/1668/ OI /7330/61 (1962)